

Neutron skin of ^{48}Ca deduced from interaction cross-section

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The neutron skin thickness of ^{48}Ca was deduced from the interaction cross-section by adopting a microscopic optical potential. The optical potential used was constructed by folding a chiral g matrix and the Skyrme mean-field densities renormalized by considering the information of the interaction cross-section. The result was $R_{\text{skin}} = 0.139 \pm 0.058$ fm.

Keywords: ^{48}Ca ; neutron skin; interaction cross-section.

The neutron skin thickness R_{skin} is not only one of the basic quantities of the structure of terrestrial nuclei, but also strongly correlated with the stiffness of the equation of state of the nucleonic matter that composes neutron stars. This can be deduced in several ways. Among them, parity-violating electron scattering¹ is thought to be the most precise means of determining the neutron root-mean-square (RMS) radii R_n , which are hardly determined by hadronic probes. In contrast, the proton RMS radii R_p were accurately determined from the elastic electron scatterings. Theoretically, only mean-field calculations are practically available for heavy nuclides. Hamiltonians or energy-density functionals adopted there contain many parameters informed by the measured quantities of representative stable and some unstable nuclides. This indicates that the neutron sector is less constrained, particularly for heavy nuclides. Mean-field calculations directly give R_n , R_p , and consequently $R_{\text{skin}} = R_n - R_p$, but the results should be critically assessed.

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A new method that relies more directly on another experimental observable, the reaction cross-section σ_R , was proposed in Refs. 2 and 3. From the experimental perspective, this is a method to extract R_{skin} from σ_R based on a reaction model with a microscopic optical potential. At the same time, from a theoretical perspective, this improves the calculated R_{skin} given by mean-field models with energy-density functionals whose parameters might not yet be fully constrained. Specifically, the adopted optical potential is constructed by folding a chiral g matrix,⁴ given by localizing the one originally constructed by taking into account the next-to-next-to-next-to leading order ($N^3\text{LO}$) two-body force and the NNLO three-body force in chiral perturbation,⁵ and Gogny/Skyrme mean-field densities. As a result, the authors of Refs. 2 and 3 obtained for ^{208}Pb , $R_{\text{skin}} = 0.278 \pm 0.035 \text{ fm}^2$ and $R_{\text{skin}} = 0.416 \pm 0.146 \text{ fm}$,³ respectively, which are consistent with PREX II with parity-violating electron scattering, $R_{\text{skin}} = 0.283 \pm 0.071 \text{ fm}$,⁶ within the error bars. Here, we should note that there still remains tension among experimental information, from PREX II, and, for example, from the elastic⁷ and the inelastic⁸ proton scatterings, the antiprotonic atom,⁹ and the coherent pion production.¹⁰

In this study, we examine ^{48}Ca by adopting a Skyrme parameter set. Reference data were obtained from Tanaka *et al.*¹¹ They measured the interaction cross-sections σ_I of Ca isotopes + ^{12}C scatterings at 280 MeV/nucleon, which are almost the same as σ_R above 100 MeV/nucleon. We compare in the following our results for R_{skin} with theirs, $R_{\text{skin}} = 0.146 \pm 0.060 \text{ fm}$, deduced using the optical limit of the Glauber model with the Woods–Saxon density. A dip in the isotope dependence was observed for ^{48}Ca , whereas the theoretical result was smooth.¹² In addition, a precision datum from the ongoing CREX project will soon be obtained.

We adopted the SLy7 parameter set, which was constructed by improving the famous SLy4 set.¹³ A Skyrme–Hartree–Fock–Bogoliubov (SHFB) calculation¹⁴ using this parameter set directly yields $(R_n, R_p, R_{\text{skin}}) = (3.600, 3.447, 0.153) \text{ fm}$. In contrast to the ^{208}Pb case,³ the third one, $R_{\text{skin}} = 0.153 \text{ fm}$, was consistent with the reference data of Tanaka *et al.*, $R_{\text{skin}} = 0.146 \pm 0.060 \text{ fm}$. In order to look into the results more closely, we consulted the precision electric scattering data adopted there, $R_p = 3.385 \text{ fm}$, and renormalized the SHFB densities to remedy possibly weak constraints on mean-field parameters.

The theoretical framework is briefly summarized here. The optical potential to determine the scattering wave function and thus the cross-section is given by folding the mean-field densities and the g matrix and consists of the direct and exchange parts

$$U^{\text{DR}}(\mathbf{R}) = \sum_{\mu,\nu} \int \rho_P^\mu(\mathbf{r}_P) \rho_T^\nu(\mathbf{r}_T) g_{\mu\nu}^{\text{DR}}(s; \rho_{\mu\nu}) d\mathbf{r}_P d\mathbf{r}_T, \quad (1)$$

$$U^{\text{EX}}(\mathbf{R}) = \sum_{\mu,\nu} \int \rho_P^\mu(\mathbf{r}_P, \mathbf{r}_P - \mathbf{s}) \rho_T^\nu(\mathbf{r}_T, \mathbf{r}_T + \mathbf{s}) \times g_{\mu\nu}^{\text{EX}}(s; \rho_{\mu\nu}) \exp[-i\mathbf{K}(\mathbf{R}) \cdot \mathbf{s}/M] d\mathbf{r}_P d\mathbf{r}_T, \quad (2)$$

where $\mathbf{s} = \mathbf{r}_P - \mathbf{r}_T + \mathbf{R}$ for the coordinate \mathbf{R} between the projectile (P) and target (T). The coordinate \mathbf{r}_P (\mathbf{r}_T) denotes the location of the interacting nucleon measured from the center of mass of P(T). Each of μ and ν stands for the z -component of isospin. Note that we use the localized version of U^{EX} . The g matrix depends on the local density at the midpoint of the interacting nucleon pair, and taken from the numerical table.¹⁵ As a theoretical detail, we note here that two steps of localization are taken in the course of obtaining the optical potential: the first one is the localization of the g matrix described in Ref. 4, and the second one is that of the folding potential described in Ref. 16.

The densities in the above potentials are renormalized as: We define the scaled density $\rho_{\text{scaling}}(\mathbf{r})$ from the original density $\rho(\mathbf{r})$ as

$$\rho_{\text{scaling}}(\mathbf{r}) = \frac{1}{\alpha^3} \rho(\mathbf{r}/\alpha) \quad (3)$$

with a scaling factor

$$\alpha = \sqrt{\frac{\langle \mathbf{r}^2 \rangle_{\text{scaling}}}{\langle \mathbf{r}^2 \rangle}}. \quad (4)$$

The actual procedures to determine α (of p and n) are as follows: First, we scale the proton density so as to be $R_p(\text{scaling}) = R_p(\text{exp})$; second, we scale the neutron density so that the calculated σ_I reproduces the data with an error bar, as shown in Fig. 1.

Double-folding calculations with renormalized densities yield $(R_n, R_{\text{skin}}) = (3.524 \pm 0.058, 0.139 \pm 0.058)$ fm. This indicates that the decreases in R_n and R_p cancel each other, and consequently R_{skin} remains similar as shown in Fig. 1. Therefore, the renormalization is necessary but the difference in the adopted reaction models does not appear here. In order to see the influence of the adopted mean field, first we checked the standard SLy4 set. The reason why we adopted the SLy7 set is that it is advertised that the SLy7 set was obtained by improving the SLy4 with respect to both a spin-gradient term and a more refined two-body center of mass correction and the joint contribution of the two terms brings significant improvement for Pb isotopes.¹³ But we found these effects on the present calculation were negligible. Second, we examined the Gogny D1S force instead of Skyrme forces. It leads to $R_{\text{skin}} = 0.105 \pm 0.06$ fm. As for the effective nucleon–nucleon interaction, only the adopted chiral g matrix is available for us. We think it reliable because it was confirmed not only in scattering calculations⁴ but also in structure calculations.¹⁷

Finally, we compare the present result with other information: Results of the high-resolution E1 polarizability experiment, 0.17 ± 0.03 fm,¹⁸ and an *ab initio* coupled-cluster calculation available for light nuclides, 0.135 ± 0.015 fm.¹⁹ We confirmed that all the results examined here fall into these ranges around 0.15 fm. By consulting the fitted correlation²⁰ between R_{skin} of ^{48}Ca and ^{208}Pb

$$R_{\text{skin}}^{48} = 0.5547 R_{\text{skin}}^{208} + 0.0718 \quad (5)$$

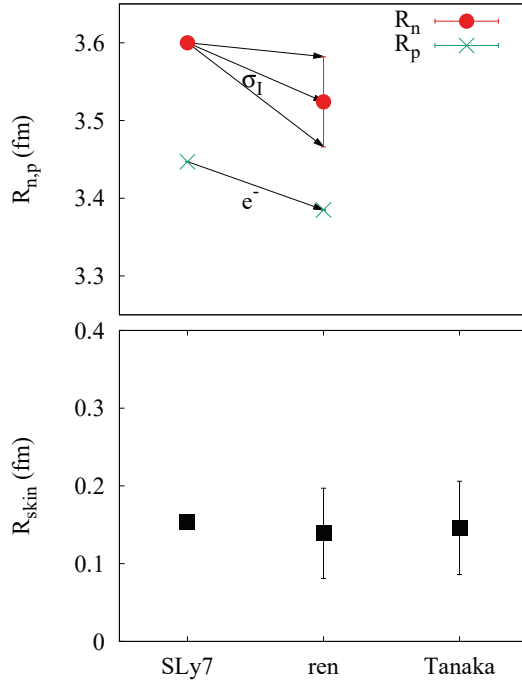


Fig. 1. Neutron and proton radii, and skin thicknesses: Directly given by a Skyrme–Hartree–Fock–Bogoliubov calculation (“SLy7”: left), deduced in the present renormalization method (“ren”: center), and given by Tanaka *et al.*¹¹ adopting the Glauber model (“Tanaka”: right). Effects of the density renormalization to reproduce the electron (e^-) scattering and the interaction cross-sections (σ_I) data are shown by arrows.

and that between R_{skin}^{208} and the slope parameter L of symmetry energy²¹

$$R_{\text{skin}}^{208} = 0.00147 L + 0.101, \quad (6)$$

this suggests rather soft slope parameters around 27 MeV, in contrast to the ^{208}Pb results of PREX II and Refs. 2 and 3. This looks to indicate that there is a room to improve the simple linear correlation (5), and its physical origin should be looked for.

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