Tilting Instability in Negative-γ Rotating Nuclei

Masayuki Matsuzaki[*)]

*Department of Physics, Fukuoka University of Education,
Munakata 811-4192, Japan*

On the basis of the cranking model and the random phase approximation, we show that the wobbling excitation on top of the s band in ^{182}Os is stable with respect to angular momentum tilting. This is consistent with the general trend that the wobbling excitations in $\gamma < 0$ rotating nuclei are more stable than those in $\gamma > 0$ ones found in our previous studies. In higher N isotopes known to be γ soft, however, a different type of tilting instability is conjectured. Its possible correspondence to experimental data is also discussed.

Symmetry breaking in nuclear mean fields is analogous to second-order phase transitions in infinite systems, and it is one of the key concepts in the theory of nuclear collective motion. According to the general concept of symmetry breaking, when one approaches a transition point from the symmetric side, a softening of collective vibrational modes takes place as a precursor to the phase transition. The following are examples: 1) in the spherical to axial shape transition, the 2^+ quadrupole vibration softens; 2) in the axial to triaxial shape transition, the γ vibration softens; and 3) in the normal fluid to superfluid transition, the pair transfer cross section increases.

Collective rotation of axially symmetric nuclei takes place only about a principal axis (usually referred to as the x axis) perpendicular to the symmetry axis (the z axis). In triaxially deformed nuclei, however, rotations about all three principal axes are possible. Therefore, if triaxiality sets in gradually, the angular momentum vector begins to wobble when viewed from the principal axis frame. Eventually, the angular momentum vector tilts permanently from the x axis. This regime is called "tilted axis rotation" (TAR), in contrast to the usual principal axis rotation (PAR). Thus, the softening of the wobbling motion is the precursor to symmetry breaking from PAR to TAR. We call this instability of the PAR mean field, caused by the softening of the wobbling motion, the "tilting instability". After the appearance of this instability, a TAR mean field, in which the signature quantum number that is associated with a rotation by π radians about the x axis is broken, replaces the PAR mean field. As shown in Eq. (1), the excitation energy of the wobbling motion is determined by moments of inertia, which represent the dynamical response of the system to rotation from a microscopic viewpoint. Therefore, not only do the moments of inertia depend on γ deformation, but also the γ deformation itself depends on the rotation frequency, since the rotational alignments of quasiparticles exert shape-driving effects on the entire system that are determined by their positions in the shell.

Small amplitude wobbling motion at high spins was first investigated by Bohr

[*)] E-mail: matsuza@fukuoka-edu.ac.jp

and Mottelson[1] in terms of a macroscopic rotor model with constant moments of inertia. Next it was studied microscopically by Janssen and Mikhailov[2] and Marshalek[3] in terms of the random phase approximation (RPA), which gives dynamical moments of inertia. Because the small amplitude wobbling mode has the same quantum numbers, parity $\pi = +$ and signature $\alpha = 1$, as the odd-spin member of the γ vibrational band, Mikhailov and Janssen[4] conjectured that it would appear as a high-spin continuation of the odd-spin γ band. However, it is not clear in which nuclei, at what spins, and with what shapes it would appear. Using the RPA, Shimizu and Matsuyanagi[5] studied Er isotopes with small $|\gamma|$, and Matsuzaki[6] and Shimizu and Matsuzaki[7] studied ^{182}Os with a rather large negative γ, but the correspondence of their results to experimental data is not clear. In 2001, Ødegård et al.[8] found an excited triaxial superdeformed (TSD) band in ^{163}Lu and identified it conclusively as a wobbling band by comparing the observed and theoretical interband $E2$ transition rates. These data were investigated using a particle-rotor model (PRM) by Hamamoto[9] and with the RPA by Matsuzaki et al.[10] In the latter work, the calculated dynamical moments of inertia depend on rotation frequency, even when the shape of the mean field is fixed. This dependence is essential for understanding the observed behavior of the excitation energy. In 2002, two-phonon wobbling excitations were also observed by Jensen et al.,[11] and their excitation energies exhibit some anharmonicity. In Ref. 10), a numerical example of the softening of the wobbling motion in a positive-γ nucleus, ^{147}Gd, is presented. Matsuzaki and Ohtsubo[12] elucidated that study by examining the shape change of the potential surface as a function of the tilting angles. In that paper, it is also discussed that the observed anharmonicity may be a signature of the onset of softening. Oi[13] proposed a new model to account for this softening. Almehed et al.[14] also discussed this. Recently, Tanabe and Sugawara-Tanabe proposed an approximation method to solve the PRM and applied it to the TSD bands.[15] Kvasil and Nazmitdinov derived a prediction for the wobbling excitations in normal deformed nuclei[16] by utilizing the sum rule type criterion found in Ref. 17).

The excitation energy of the wobbling motion is given, as a function of the moments of inertia, by[1]

$$\hbar\omega_{\text{wob}} = \hbar\omega_{\text{rot}}\sqrt{\frac{(\mathcal{J}_x - \mathcal{J}_y)(\mathcal{J}_x - \mathcal{J}_z)}{\mathcal{J}_y\mathcal{J}_z}} \ , \tag{1}$$

where ω_{rot} is the frequency of the main rotation about the x axis and $\mathcal{J}_x$, $\mathcal{J}_y$ and $\mathcal{J}_z$ are the moments of inertia about the three principal axes. This implies that $\mathcal{J}_x > \mathcal{J}_y, \mathcal{J}_z$ or $\mathcal{J}_x < \mathcal{J}_y, \mathcal{J}_z$ must be satisfied for ω_{wob} to be real. The irrotational model moment of inertia is given by

$$\mathcal{J}_k^{\text{irr}} \propto \sin^2\left(\gamma + \frac{2}{3}\pi k\right), \tag{2}$$

with $k = 1$, 2 and 3 denoting the x, y and z principal axes, respectively, and its γ *dependence* is believed to be realistic. If we use this form of $\mathcal{J}_k^{\text{irr}}$, $-60° < \gamma < 0$ for the former or $-120° < \gamma < -90°$ or $30° < \gamma < 60°$ for the latter is required. Because

the γ deformation of the observed TSD band is $\gamma \sim +20°$, another mechanism is necessary for the wobbling excitation to exist. It was found in Ref. 10) and elucidated in Ref. 17) that the alignment of the last odd quasiproton imparts an additional contribution to $\mathcal{J}_x$ and consequently makes $\mathcal{J}_x > \mathcal{J}_y$ in place of $\mathcal{J}_x < \mathcal{J}_y$ in the irrotational-like behavior. But the smallness of the quantity $\mathcal{J}_x - \mathcal{J}_y$ implies that the excitation is fragile.

The negative-γ collective rotation, $-60° < \gamma < 0$, is expected to dominate. However, it seems difficult to excite a wobbling mode in the ground band of even-even nuclei, because $\mathcal{J}_x \sim \mathcal{J}_y$ in such cases (see Ref. 6)). Therefore, the following three conditions are desirable for the wobbling excitation to exist: 1) $-60° < \gamma < 0$; 2) $|\gamma|$ is not small; and 3) the existence of aligned quasiparticle(s) that make $\mathcal{J}_x$ larger. From these conditions, we chose the s band of ^{182}Os as a representative in Refs. 6) and 7). There, we concluded that a wobbling excitation exists on top of the s band of ^{182}Os. Recently, Hashimoto and Horibata[18] reached the opposite conclusion. Here, we briefly comment on their work before proceeding to the main discussion of this paper. They recently presented a renewed three-dimensional cranking calculation for ^{182}Os, considering the stability of the s band on the basis of their previous calculation.[19] They concluded that the wobbling excitation in the s band does not exist; this contradicts the result of our previous calculation.[6,7] Closely examining their works, it is found that the s band that they study has different properties from that which we study. Although not stated in Ref. 18), it is reported in Ref. 19) that their s band consists of two aligned *quasiprotons*. Their low-Ω $h_{9/2}$ character would lead to a positive-γ shape. Note that their convention for the sign of γ is opposite to the Lund convention adopted here. As stated above, wobbling excitations in positive-γ nuclei are fragile. Although our calculation uses fixed mean field parameters, we took account of the experimental information, which suggests that the s band consists of two aligned $i_{13/2}$ *quasineutrons*.[20] Because the Fermi surface is located at a high position in the $i_{13/2}$ shell, alignment leads to a negative-γ shape. As discussed above, the wobbling excitations in negative-γ quasiparticle aligned configurations are rather stable. This is the reason that the conclusions of Hashimoto and Horibata differ from ours. Collective excitation in the g band is expected or exists in both calculations, but in our calculation, it is γ vibration-like rather than wobbling-like (see Ref. 4)).

Now we proceed to present a numerical example of another type of tilting instability in negative-γ rotating nuclei, which is different from that in the positive-γ cases studied in our previous works,[10,12] although wobbling excitations are stable in many negative-γ cases, when they exist. (The meaning of the term "different" used here is elucidated below.) First, we briefly review our model. We begin with a one-body Hamiltonian in the rotating frame,

$$h' = h - \hbar\omega_{\mathrm{rot}}J_x, \tag{3}$$

$$h = h_{\mathrm{Nil}} - \Delta_\tau(P_\tau^\dagger + P_\tau) - \lambda_\tau N_\tau, \tag{4}$$

$$h_{\mathrm{Nil}} = \frac{\boldsymbol{p}^2}{2M} + \frac{1}{2}M(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) + v_{ls}\boldsymbol{l} \cdot \boldsymbol{s} + v_{ll}(\boldsymbol{l}^2 - \langle \boldsymbol{l}^2 \rangle_{N_{\mathrm{osc}}}). \tag{5}$$

In Eq. (4), $\tau = 1$ and 2 indicate a neutron and proton, respectively, and the chemical

potentials λ_τ are determined so as to give the correct average particle numbers $\langle N_\tau \rangle$. The oscillator frequencies in Eq. (5) are related to the quadrupole deformation parameters ϵ_2 and γ in the usual way. They, along with the pairing gaps, Δ_τ, are treated as parameters. The orbital angular momentum $\boldsymbol{l}$ in Eq. (5) is defined in the singly stretched coordinates $x'_k = \sqrt{\frac{\omega_k}{\omega_0}} x_k$ and the corresponding momenta, with $k = 1, 2$ and 3 denoting x, y and z, respectively. Since h' conserves the parity, π, and the signature, α, nuclear states can be labeled by them. We apply the RPA to the residual pairing plus doubly stretched quadrupole-quadrupole $(Q'' \cdot Q'')$ interaction between quasiparticles. Because we are interested in wobbling motion that has a definite signature quantum number, $\alpha = 1$, only two components out of five of the $Q'' \cdot Q''$ interaction are relevant. They are given by

$$H_{\text{int}}^{(-)} = -\frac{1}{2} \sum_{K=1,2} \kappa_K^{(-)} Q_K''^{(-)\dagger} Q_K''^{(-)}, \tag{6}$$

where the doubly stretched quadrupole operators are defined by

$$Q_K'' = Q_K \left(x_k \rightarrow x_k'' = \frac{\omega_k}{\omega_0} x_k \right), \tag{7}$$

and those with good signature are

$$Q_K^{(\pm)} = \frac{1}{\sqrt{2(1 + \delta_{K0})}} \left(Q_K \pm Q_{-K} \right). \tag{8}$$

The residual pairing interaction does not contribute, because P_τ is an operator with $\alpha = 0$. The equation of motion,

$$\left[h' + H_{\text{int}}^{(-)}, X_n^\dagger \right]_{\text{RPA}} = \hbar \omega_n X_n^\dagger, \tag{9}$$

for the eigenmode

$$X_n^\dagger = \sum_{\mu < \nu}^{(\alpha = \pm 1/2)} \left(\psi_n(\mu\nu) a_\mu^\dagger a_\nu^\dagger + \varphi_n(\mu\nu) a_\nu a_\mu \right) \tag{10}$$

leads to a pair of coupled equations for the transition amplitudes:

$$T_{K,n} = \left\langle \left[Q_K^{(-)}, X_n^\dagger \right] \right\rangle. \tag{11}$$

Then, assuming $\gamma \neq 0$, this can be cast[3] into the form

$$(\omega_n^2 - \omega_{\text{rot}}^2) \left[\omega_n^2 - \omega_{\text{rot}}^2 \frac{\left(\mathcal{J}_x - \mathcal{J}_y^{(\text{eff})}(\omega_n) \right) \left(\mathcal{J}_x - \mathcal{J}_z^{(\text{eff})}(\omega_n) \right)}{\mathcal{J}_y^{(\text{eff})}(\omega_n) \mathcal{J}_z^{(\text{eff})}(\omega_n)} \right] = 0. \tag{12}$$

This expression implies that the spurious mode (corresponding to $\omega_n = \omega_{\text{rot}}$, which is not a real intrinsic excitation but an overall rotation), represented by the first

factor, and all normal modes, represented by the second factor, are decoupled. Here $\mathcal{J}_x = \langle J_x \rangle / \omega_{\text{rot}}$, as usual, and detailed expressions of $\mathcal{J}_{y,z}^{(\text{eff})}(\omega_n)$ are given in Refs. 3), 6) and 7). Among normal modes, we obtain

$$\omega_{\text{wob}} = \omega_{\text{rot}} \sqrt{\frac{\left(\mathcal{J}_x - \mathcal{J}_y^{(\text{eff})}(\omega_{\text{wob}})\right)\left(\mathcal{J}_x - \mathcal{J}_z^{(\text{eff})}(\omega_{\text{wob}})\right)}{\mathcal{J}_y^{(\text{eff})}(\omega_{\text{wob}})\mathcal{J}_z^{(\text{eff})}(\omega_{\text{wob}})}}, \tag{13}$$

by setting $\omega_n = \omega_{\text{wob}}$. Note that this gives a real excitation only when the argument of the square root is positive, and it is a non-trivial problem to find whether or not a collective solution appears. Evidently, this coincides with the form (1) derived by Bohr and Mottelson for a rotor model[1] and is known in classical mechanics.[21] Further, this makes it possible to describe the mechanism of the tilting instability in terms of the dynamical moments of inertia. The wobbling angles that measure the amplitude of the vibrational motion of the angular momentum vector around the x axis are defined by

$$\theta_{\text{wob}} = \tan^{-1} \frac{\sqrt{|J_y^{(\text{PA})}(\omega_{\text{wob}})|^2 + |J_z^{(\text{PA})}(\omega_{\text{wob}})|^2}}{\langle J_x^{(\text{PA})} \rangle}, \tag{14}$$

$$\varphi_{\text{wob}} = \tan^{-1} \left| \frac{J_z^{(\text{PA})}(\omega_{\text{wob}})}{J_y^{(\text{PA})}(\omega_{\text{wob}})} \right|, \tag{15}$$

with (PA) indicating the principal axis frame. The PA components of the angular momentum vector are defined by

$$\langle J_x^{(\text{PA})} \rangle = \langle J_x \rangle, \tag{16}$$

$$i J_y^{(\text{PA})} = i J_y - \frac{\langle J_x \rangle}{2 \langle Q_2^{(+)} \rangle} Q_2^{(-)}, \tag{17}$$

$$J_z^{(\text{PA})} = J_z - \frac{\langle J_x \rangle}{\sqrt{3} \langle Q_0^{(+)} \rangle - \langle Q_2^{(+)} \rangle} Q_1^{(-)} \tag{18}$$

in terms of the RPA matrix elements of their uniformly rotating frame components usually calculated in the cranking model,[3),6),7)] because the PA frame is determined by diagonalizing the quadrupole tensor $Q_K^{(\text{PA})}$.[3),22)]

We choose ^{186}Os, considering the possible correspondence to the experimental data. The s band consists of $(\nu i_{13/2})^2$. In this calculation, we concentrate on the direct rotational effect, ignoring the effect of the possible rotational shape change. The adopted mean field parameters are $\epsilon_2 = 0.205$, $\gamma = -32°$, and $\Delta_n = \Delta_p = 0.4$ MeV. Calculations were performed in the model space of five major shells: $N_{\text{osc}} = 3 - 7$ for neutrons and $2 - 6$ for protons. The strengths of the $\boldsymbol{l} \cdot \boldsymbol{s}$ and $\boldsymbol{l}^2$ potentials were taken from Ref. 23). Figure 1 plots the excitation energy $\hbar \omega_{\text{wob}}$ in the rotating frame. The decrease of this quantity indicates the instability of the principal axis rotating s band that supports the small amplitude wobbling excitation. Figure 2 displays the wobbling angles θ_{wob} and φ_{wob}. It is seen that, while the angular
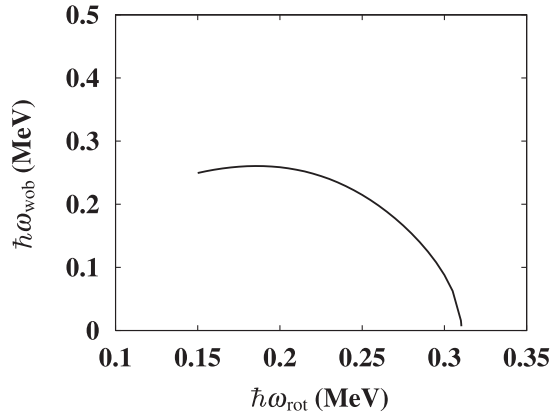
Fig. 1.   Rotation frequency dependence of the wobbling excitation energy in the s band of ^{186}Os.

momentum vector wobbles around the x axis with $\theta_{\mathrm{wob}} \simeq 15°$ up to just below the instability point, φ_{wob} increases gradually. This means that the z component increases gradually. Eventually, at the instability point, the angles appear to reach $\theta_{\mathrm{wob}} > 45°$ and $\varphi_{\mathrm{wob}} = 90°$, that is, the angular momentum vector tilts to the x-z plane. Although the present calculation cannot go beyond the instability point, a numerical investigation of the correspondence between the instability of the PAR and the TAR that follows it is presented in Ref. 12). More direct information concerning the shape that the system would favor can be obtained from the moments of inertia, displayed in Fig. 3. This figure shows that $\mathcal{J}_x = \mathcal{J}_z$ is realized at the instability point; this is a *different* type of tilting instability from that observed in $\gamma > 0$ nuclei, which is caused by $\mathcal{J}_x = \mathcal{J}_y$. Here, we elucidate the meaning of "*different* type". The instability brought about by the condition $\mathcal{J}_x = \mathcal{J}_y$, discussed in Refs. 10) and 12), and that by the condition $\mathcal{J}_x = \mathcal{J}_z$, discussed here, are similar in the sense that the energy costs of rotations about two different axes coincide. However, here we base our discussion on the physical picture within which $\gamma > 0$ and $\gamma < 0$ represent different rotation schemes and, in accordance with the fact discussed above that for $\gamma > 0$, ω_{wob} cannot be real without aligned quasiparticle that makes $\mathcal{J}_x$ larger, while for $\gamma < 0$, ω_{wob} can be real without one. Note that nothing peculiar happens at $\mathcal{J}_y = \mathcal{J}_z$, because the instability is given by the zeros of Eq. (13). Although a self-consistent shape change is beyond the scope of the present simple-minded calculation, the relation $\mathcal{J}_x = \mathcal{J}_z$ may indicate that either a TAR ($\mathcal{J}_y \neq 0$) or another PAR, that is, an oblate collective rotation ($\mathcal{J}_y = 0$ for the irrotational rotor), would be favored. The possibility of oblate collective rotation was first investigated by Hilton and Mang[24] for ^{180}Hf, and very recently by Walker and Xu[25] and Sun et al.[26] for ^{190}W. In the present case, $\mathcal{J}_y$ is decreasing, but not 0. Therefore, it is natural to regard the rotation scheme just after the instability as a TAR.

Although elucidating the quantitative criterion for the occurrence of the instability is beyond the scope of the present calculation, we confirmed that an instability occurs at lower rotation frequencies for smaller ϵ_2 or larger N. These results seem to indicate the consistency with the N dependence of the γ softness in this mass region
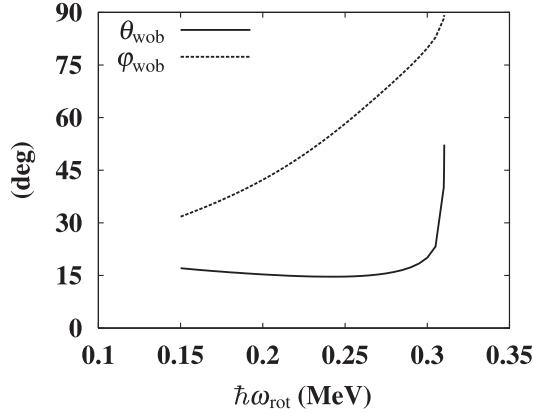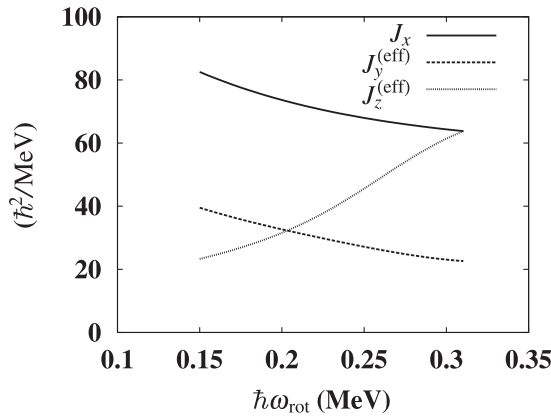
Fig. 2. Rotation frequency dependence of the wobbling angles.

Fig. 3. Rotation frequency dependence of the moments of inertia.

seen in quadrupole deformations,[27] the excitation energy of the γ vibration,[1] and high-K isomerism.[28]

Finally, we mention the possible correspondence to the observed data. In Ref. 29), Balabanski et al. reported an anomalous termination of the yrast band of ^{186}Os at 18^+. According to their calculation, the $(\nu i_{13/2})^2$ alignment drives the shape to $\gamma \simeq -30°$ before this termination. Actually, the mean field parameters of the present calculation were chosen so as to conform with this behavior. With respect to the termination itself, they argued, using a total Routhian surface calculation, that it is related to a further shape change in the γ direction. Later, Wheldon et al.[28] asserted that it does not terminate. Aside from the different conclusions concerning the fate of the higher spin states, the character of the yrast band is found to change at 14^+ in both studies. Wheldon et al.[28] concluded that this is caused by the crossing with the tilted 10^+ band. Because the main component of the high spin part of the ground state (gs) band is thought to be a PAR triaxial $(\nu i_{13/2})^2$ s band,[30] the observed crossing is qualitatively attributed to the instability of the PAR mean field. On the

other hand, because the 14^+ and 12^+ members of the gs band correspond to $\hbar\omega_{\mathrm{rot}} = 0.389$ MeV and 0.356 MeV, respectively, the observed crossing takes place between them. Therefore, quantitatively, the result of the present calculation, according to which it takes place at around $\hbar\omega_{\mathrm{rot}} = 0.310$ MeV, is not fully consistent with the experimental results.

To summarize, in this paper, first we pointed out that the wobbling excitations in $\gamma < 0$ quasiparticle aligned bands should be more stable than those in $\gamma > 0$ ones, as found in our previous studies. In relation to this, we have clarified the reason for the different conclusions regarding the existence of the wobbling excitation on top of the s band of ^{182}Os reached in the recent work of Hashimoto and Horibata[18] and in ours. Second, we showed that, in spite of this difference, the wobbling excitation in $\gamma < 0$ nuclei at higher N can become unstable by presenting a numerical example, although elucidation of the quantitative criterion for the occurrence of this type of tilting instability is deferred to more elaborate calculations. The possible correspondence of this example to the experimental data was also discussed.

References

1) A. Bohr and B. R. Mottelson, *Nuclear Structure Vol. II* (Benjamin, New York, 1975).
2) D. Janssen and I. N. Mikhailov, Nucl. Phys. A **318** (1979), 390.
3) E. R. Marshalek, Nucl. Phys. A **331** (1979), 429.
4) I. N. Mikhailov and D. Janssen, Phys. Lett. B **72** (1978), 303.
5) Y. R. Shimizu and K. Matsuyanagi, Prog. Theor. Phys. **72** (1984), 799.
6) M. Matsuzaki, Nucl. Phys. A **509** (1990), 269.
7) Y. R. Shimizu and M. Matsuzaki, Nucl. Phys. A **588** (1995), 559.
8) S. W. Ødegård et al., Phys. Rev. Lett. **86** (2001), 5866.
9) I. Hamamoto, Phys. Rev. C **65** (2002), 044305.
10) M. Matsuzaki, Y. R. Shimizu and K. Matsuyanagi, Phys. Rev. C **65** (2002), 041303(R).
11) D. R. Jensen et al., Phys. Rev. Lett. **89** (2002), 142503.
12) M. Matsuzaki and S.-I. Ohtsubo, Phys. Rev. C **69** (2004), 064317.
13) M. Oi, Phys. Lett. B **634** (2006), 30.
14) D. Almehed, R. G. Nazmitdinov and F. Dönau, Physica Scripta **T125** (2006), 139.
15) K. Tanabe and K. Sugawara-Tanabe, Phys. Rev. C **73** (2006), 034305.
16) J. Kvasil and R. G. Nazmitdinov, Phys. Lett. B **650** (2007), 331.
17) M. Matsuzaki, Y. R. Shimizu and K. Matsuyanagi, Phys. Rev. C **69** (2004), 034325.
18) Y. Hashimoto and T. Horibata, Phys. Rev. C **74** (2006), 017301.
19) T. Horibata and N. Onishi, Nucl. Phys. A **596** (1996), 251.
20) T. Kutsarova et al., Nucl. Phys. A **587** (1995), 111.
21) L. D. Landau and E. M. Lifshitz, *Mechanics* (Pergamon, London, 1960).
22) A. K. Kerman and N. Onishi, Nucl. Phys. A **361** (1981), 179.
23) T. Bengtsson and I. Ragnarsson, Nucl. Phys. A **436** (1985), 14.
24) R. R. Hilton and H. J. Mang, Phys. Rev. Lett. **43** (1979), 1979.
25) P. M. Walker and F. R. Xu, Phys. Lett. B **635** (2006), 286.
26) Y. Sun, P. M. Walker, F. R. Xu and Y. X. Liu, Phys. Lett. B, in press; arXiv:0711.0260.
27) D. Cline, Annu. Rev. Nucl. Part. Sci. **36** (1986), 683.
28) C. Wheldon et al., Phys. Rev. C **59** (1999), R2334.
29) D. L. Balabanski et al., Z. Phys. A **356** (1997), 393.
30) R. Wyss, W. Satula, W. Nazarewicz and A. Johnson, Nucl. Phys. A **511** (1990), 324.