Structure of strange dwarfs with colour superconducting core

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Abstract

We study the effects of two-flavour colour superconductivity on the structure of strange dwarfs, which are stellar objects with similar masses and radii with ordinary white dwarfs but stabilized by the strange quark matter core. We find that unpaired quark matter is a good approximation to the core of strange dwarfs.

Witten made a conjecture that the absolute ground state of quantum chromodynamics (QCD) is not ^{56}Fe but strange quark matter, which is a plasma composed of almost an equal number of deconfined u, d and s quarks [1]. Although this conjecture has been neither confirmed nor rejected, if this is true, since deconfinement is expected in high density cores of compact stars, there could exist stars that contain strange quark matter converted from two-flavour quark matter via weak interaction. Strange quark stars whose radii are about 10 km, with or without a *thin* nuclear crust, have long been investigated.

Glendenning *et al* proposed a new class of compact stars containing strange quark matter and *thick* nuclear crust ranging from a few hundred to 10 000 km [2–4]. They named them the strange dwarfs because their radii correspond to those of white dwarfs. Alcock *et al* discussed the mechanism that the strange quark core supports the crust [5]. Since the mass of s quark is larger than that of u and d, strange quark matter is positively charged. In order to electrically neutralize the core, electrons are bound to the surface of the core. They estimated that the thickness of this electric dipole layer is a few hundred fm. Then this layer can support a nuclear crust. Although Alcock *et al* considered only thin crusts, Glendenning *et al* considered thick crusts up to about 10 000 km. Very recently, Mathews *et al* identified eight candidates of strange dwarfs from observed data [6].

A theoretical facet whose importance in nuclear physics was recognized later is colour superconductivity in quark matter. At asymptotically high density, the colour-flavour locking (CFL) is believed to be the ground state [7]. At realistic densities, however, the two-flavour colour superconductivity (2SC) is thought to be realized even when electric neutrality is imposed if the coupling constant is strong [8]. Thus, in the present paper, we discuss the effects of the 2SC phase in the strange quark matter core on the structure of strange dwarfs.

In order to determine the structure of compact stars, we solve the general relativistic Tolman–Oppenheimer–Volkoff (TOV) equation,

$$\frac{\mathrm{d}p(r)}{\mathrm{d}r} = -\frac{G\epsilon(r)M(r)}{c^2 r^2}\left(1 + \frac{p(r)}{\epsilon(r)}\right)\left(1 + \frac{4\pi r^3 p(r)}{M(r)c^2}\right)\left(1 - \frac{2GM(r)}{c^2 r}\right)^{-1}, \tag{1}$$

$$M(r) = 4\pi \int_0^r \frac{\epsilon(r')}{c^2} r'^2 \mathrm{d}r', \tag{2}$$

for the pressure $p(r)$, the energy density $\epsilon(r)$ and the mass enclosed within the radius r, $M(r)$. Here, G is the gravitational constant and c is the speed of light. The equation is closed when an equation of state (EOS), a relation between p and ϵ, is specified. In the present case, strange dwarfs are composed of the strange quark matter core and the nuclear crust. Accordingly two parameters, the pressure at the centre and at the core-crust boundary, must be specified to integrate the TOV equation. The latter must be equal or less than that corresponds to the nucleon drip density ϵ_{drip}. Otherwise neutrons drip and gravitate to the core. In the present calculation, we take a p_{crust} calculated from $\epsilon_{\mathrm{crust}} = \epsilon_{\mathrm{drip}}$.

We assume zero temperature throughout this paper. As for the EOS of the quark core, we adopt the MIT bag model without any QCD corrections (see [4], for example). For unpaired free quark matter,

$$p = -B + \sum_f \frac{1}{4\pi^2}\left[\mu_f k_{\mathrm{F}f}\left(\mu_f^2 - \frac{5}{2}m_f^2\right) + \frac{3}{2}m_f^4 \ln\left(\frac{\mu_f + k_{\mathrm{F}f}}{m_f}\right)\right], \tag{3}$$

$$\epsilon = B + \sum_f \frac{3}{4\pi^2}\left[\mu_f k_{\mathrm{F}f}\left(\mu_f^2 - \frac{1}{2}m_f^2\right) - \frac{1}{2}m_f^4 \ln\left(\frac{\mu_f + k_{\mathrm{F}f}}{m_f}\right)\right], \tag{4}$$

where m_f, $k_{\mathrm{F}f}$ and $\mu_f = \sqrt{m_f^2 + k_{\mathrm{F}f}^2}$ are the mass, the Fermi momentum and the chemical potential of quarks of each flavour, respectively, and f runs u, d and s. Hereafter, we put $c = \hbar = 1$. The quantity B is the bag constant. The effect of colour superconductivity is incorporated as a chemical-potential-dependent effective bag constant. In the 2SC case [9],

$$B_{\mathrm{eff}} = B - \frac{1}{\pi^2}\Delta^2(\mu)\mu^2, \tag{5}$$

where $\Delta(\mu)$ is the quark pairing gap as a function of a chemical potential μ, whose relation to μ_f is specified later.

The pairing gap is obtained as a function of the Fermi momentum by solving the gap equation [10]

$$\Delta(k_{\mathrm{F}}) = -\frac{1}{8\pi^2}\int_0^\infty \bar{v}(k_{\mathrm{F}}, k)\frac{\Delta(k)}{E'(k)}k^2\,\mathrm{d}k, \tag{6}$$

$$E'(k) = \sqrt{(E_k - E_{k_{\mathrm{F}}})^2 + 3\Delta^2(k)}, \tag{7}$$

with $k_{\mathrm{F}} = k_{\mathrm{Fu}} = k_{\mathrm{Fd}}$, $E_k = \sqrt{k^2 + m_q^2}$ and $m_q = m_{\mathrm{u}} = m_{\mathrm{d}}$. The one gluon exchange pairing interaction is given by

$$\bar{v}(p, k) = -\frac{\pi}{3}\alpha_{\mathrm{s}}\frac{1}{pkE_pE_k}\left(\left(2E_pE_k + 2m_q^2 + p^2 + k^2 + m_{\mathrm{E}}^2\right)\ln\left(\frac{(p+k)^2 + m_{\mathrm{E}}^2}{(p-k)^2 + m_{\mathrm{E}}^2}\right)\right.$$
$$\left. + 2\left(6E_pE_k - 6m_q^2 - p^2 - k^2\right)\ln\left|\frac{p+k}{p-k}\right|\right),$$

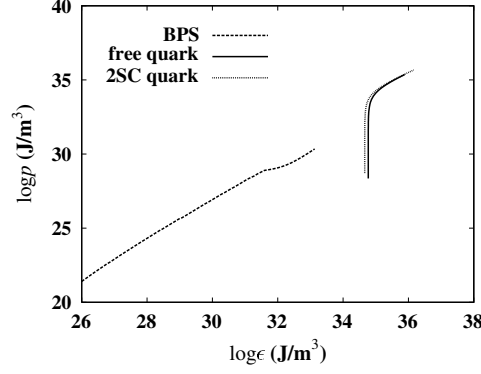$$m_{\mathrm{E}}^2 = \frac{4}{\pi}\alpha_{\mathrm{s}}\mu^2, \tag{8}$$

Figure 1. Equations of state of free and 2SC quark matter and β-equilibrium nuclear matter. The latter is tabulated in [12] and [4].
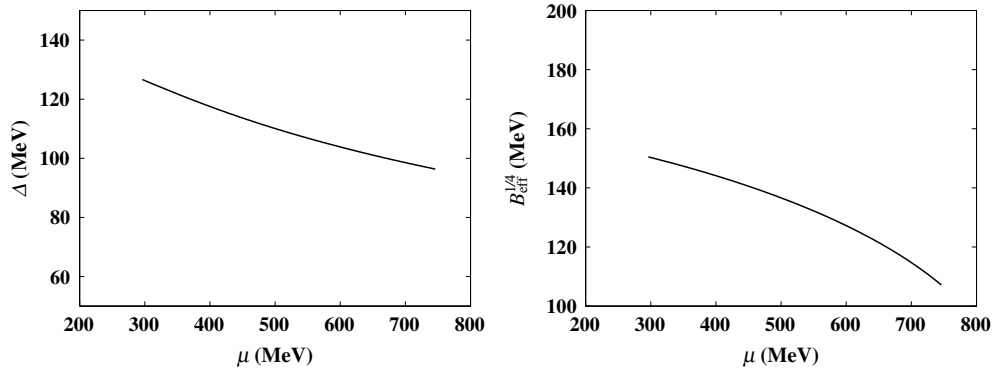

Figure 2. Left: colour superconducting pairing gap and right: effective bag constant, as functions of the quark chemical potential.

where p and k are the magnitudes of 3-momenta. The running coupling constant is given by [11]

$$\alpha_s(\mathbf{q}^2) = \frac{4\pi}{9} \frac{1}{\ln\left(\frac{q_{max}^2 + q_c^2}{\Lambda_{QCD}^2}\right)}, \qquad \mathbf{q} = \mathbf{p} - \mathbf{k}, \qquad q_{max} = \max\{p, k\}. \qquad (9)$$

As for the EOS of the crust, we adopt the tabulated one for β-equilibrium nuclear matter of Baym, Pethick and Sutherland [12] (BPS) conforming to [2–4].

The positively charged strange quark matter in the core is simply approximated by $\mu = \mu_u = \mu_d = \mu_s$. Quark masses are given by $m_u = m_d = 10$ MeV, $m_s = 150$ MeV. The bag constant is chosen to be $B^{1/4} = 160$ MeV. Parameters entering into the pairing interaction are $q_c^2 = 1.5\,\Lambda_{QCD}^2$ and $\Lambda_{QCD} = 400$ MeV. The nucleon drip density is $\epsilon_{drip} = 4.3 \times 10^{11}$ g cm^{-3}.

The adopted EOS is displayed in figure 1. The logarithm is to base 10 throughout this paper. The quark matter EOS describes the core and the BPS EOS describes the crust. At the boundary, the pressure is common whereas the energy density jumps discontinuously. In order to obtain the EOS for 2SC matter, the pairing gap must be calculated at each k_F beforehand. This is shown in figure 2(left). The effective bag constant determined by the pairing gap is shown in figure 2(right). The resulting 2SC EOS is included in figure 1.
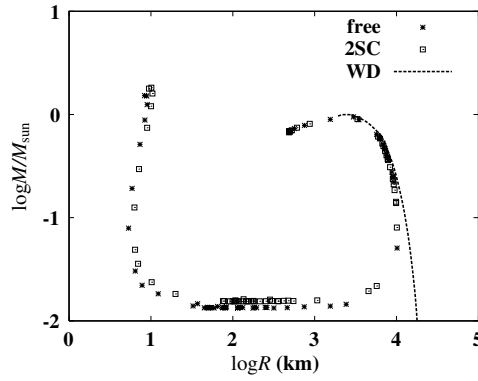
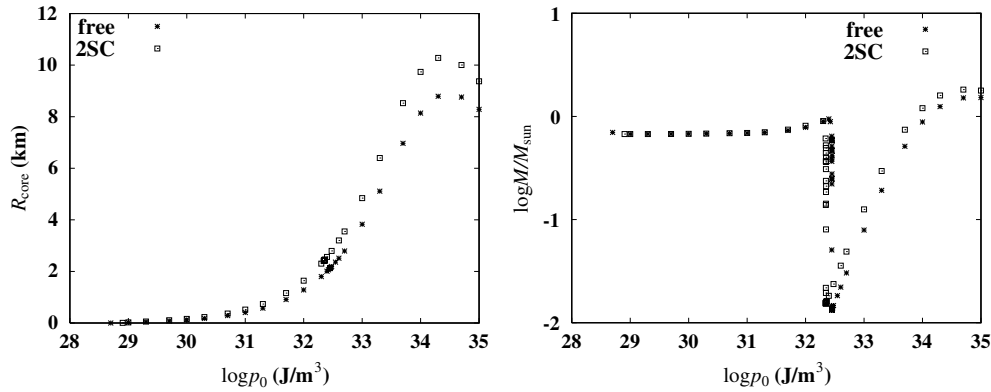**Figure 3.** Mass–radius relation of strange dwarfs and white dwarfs.

Figure 4. Left: core radius and right: mass of strange dwarfs, as functions of the central pressure.

Figure 3 presents the mass–radius relation obtained by integrating the TOV equation with a fixed p_{crust}, determined from $\epsilon_{\text{crust}} = \epsilon_{\text{drip}}$, and various central pressures. This result can be classified into three regions. The first region (larger central pressures), almost a vertical curve at around $R \sim 10$ km, describes strange stars with thin crusts. In this region, colour superconductivity makes the maximum mass and radius larger because the pairing gap reduces the bag constant, and consequently the energy density decreases and the pressure increases. This is consistent with another calculation with the CFL phase [13]. The second region, horizontal at around $M/M_{\text{sun}} \sim 10^{-2}$, and the third region, vertical at around $R \sim 10^4$ km up to the maximum mass, correspond to strange dwarfs. In the second region, colour superconducting quark cores support slightly larger masses than unpaired free quark cores. In the third region, the effect of colour superconductivity is negligible. In figure 3, the mass–radius relation of ordinary white dwarfs without quark matter cores calculated by adopting the BPS EOS is also shown although it is known that the BPS EOS is not very suitable for white dwarfs. As the central pressure decreases, the quark matter core shrinks (figure 4(left)) and eventually strange dwarfs reduce to ordinary white dwarfs. When their masses are the same, the former is more compact than the latter (see also figure 5(right)) because of the gravity of the core. Mathews *et al* paid attention to this difference in the mass–radius relation and classified the observed data of dwarfs [6]. According to their work, eight of them are classified into strange dwarfs.
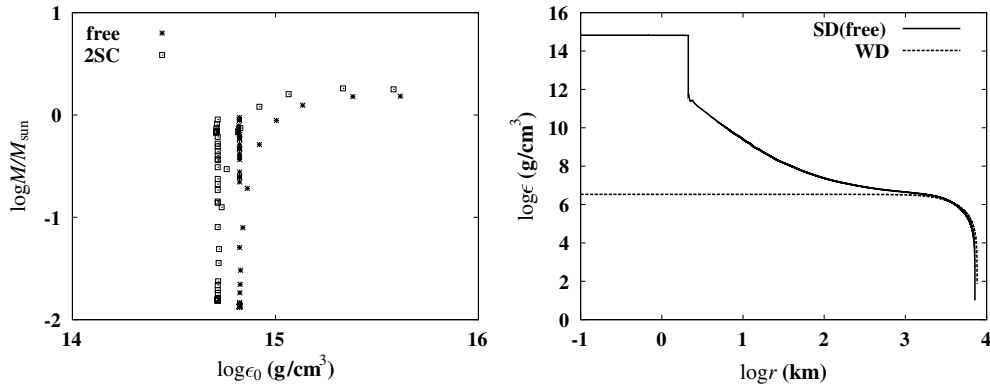
Figure 5. Left: mass of strange dwarfs as a function of the central energy density. Right: energy profile of a strange dwarf with $M/M_{\text{sun}} = 0.465$ and that of a white dwarf with $M/M_{\text{sun}} = 0.466$.

Figure 4(right) indicates that strange dwarfs, in particular those of 10^3 km $< R < 10^4$ km, are realized in a very narrow range of the central pressure. This is reflected in the density of calculated points. During this rapid structure change from the second to the third region, the core radius almost does not change; see figure 4(left). Figure 5(left) also graphs M/M_{sun} as figure 4(right), but as a function of the central energy density. The difference between these two figures at the low pressure/energy density side can be understood from the quark matter EOS in figure 1 such that the pressure decreases steeply at the lowest energy density. Figure 5(left) indicates that strange dwarfs have central energy densities just below the lowest stable compact strange stars and several orders of magnitude larger than those of ordinary white dwarfs. This is clearly demonstrated in figure 5(right).

To summarize, we have solved the Tolman–Oppenheimer–Volkoff equation for strange dwarfs with $\epsilon_{\text{crust}} = \epsilon_{\text{drip}}$ and a wide range of the central pressure. We have examined effects of the two-flavour colour superconductivity in the strange quark matter core in a simplified manner. The obtained results indicate that, aside from a slight increase of the minimum mass, the effect of colour superconductivity is negligible in the mass–radius relation. This is consistent with the conjecture given in [6]. As a function of the central energy density, however, strange dwarfs are realized at slightly lower energy densities than the unpaired free quark case reflecting the effect on the equation of state. Recently, Usov discussed that electric fields are also generated on the surface of the colour-flavour locked matter [14]. This suggests that strange dwarfs with colour-flavour locked cores might also be possible although this is expected only at relatively high densities. Since the pairing gap enters into the calculation only through the effective bag constant, aside from a possible slight change in chemical potentials, it can surely be expected that the effect of colour-flavour locking does not differ much from that of the two-flavour colour superconductivity. In conclusion, unpaired quark matter is a good approximation to the core of strange dwarfs. Another aspect that might be affected by colour superconductivity is the cooling [15]. This is beyond the scope of the present study.

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