Dynamical Moments of Inertia and Wobbling Motions in Triaxial Superdeformed Nuclei

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Abstract. We discuss some characteristic features of the wobbling motion excited on the triaxial superdeformed Lu and Hf nuclei. We show these features are determined by the behavior of the moments of inertia.

One of the most striking findings in the recent high spin spectroscopy is the discovery of the one- and two-phonon excitations of the nuclear wobbling motion. The wobbling motion is a spinning motion of the asymmetric top, where the body rotates non-uniformly about a non-inertia axis. Namely, it is a genuine three-dimensional rotational motion, and the non-axiality of nuclear moments of inertia is essential.

The total routhian surface calculation predicts the yrast triaxial superdeformation (TSD) in the neutron deficient Lu and Hf nuclei at $\varepsilon_2 \simeq 0.4$ and $\gamma \simeq +20°$ [1]. When the irrotational model moments of inertia is taken, which is given by

$$\mathscr{J}_k^{\rm irr} = 4B\beta^2 \sin^2(\gamma + \frac{2}{3}\pi k), \tag{1}$$

with $k = 1-3$ denoting the $x-z$ principal axes, B being the irrotational mass parameter, and which is believed to approximate, aside from the magnitude, the γ dependence of the moments of inertia for the nuclear collective rotation, this shape leads to $\mathscr{J}_y > \mathscr{J}_x > \mathscr{J}_z$. It seems natural and supported by the transition quadrupole moment measurement [2] to regard the observed wobbling motion as excited on the yrast TSD with that shape. Its excitation energy in the Bohr-Mottelson model is given by [3]

$$\hbar\omega_{\rm wob} = \hbar\omega_{\rm rot}\sqrt{\frac{(\mathscr{J}_x - \mathscr{J}_y)(\mathscr{J}_x - \mathscr{J}_z)}{\mathscr{J}_y \mathscr{J}_z}}. \tag{2}$$

This raises a puzzle since $\mathscr{J}_y > \mathscr{J}_x > \mathscr{J}_z$ leads to an imaginary $\omega_{\rm wob}$. The key to solve this puzzle is to consider the moment of inertia of the whole system; not only the 0 quasiparticle (QP) rotor part but also the aligned QP(s).

The cranking model plus random phase approximation (RPA) is a framework that treats the rotor part and aligned QPs on the same footing. Since the wobbling mode is an eigenmode with signature $\alpha = 1$, two out of five components of the residual $Q'' \cdot Q''$ interaction are relevant. Having decoupled the spurious mode, the RPA equation of motion is cast into the form [4]

$$\hbar\omega_{\rm wob} = \hbar\omega_{\rm rot}\sqrt{\frac{\left(\mathscr{J}_x - \mathscr{J}_y^{\rm (eff)}(\omega_{\rm wob})\right)\left(\mathscr{J}_x - \mathscr{J}_z^{\rm (eff)}(\omega_{\rm wob})\right)}{\mathscr{J}_y^{\rm (eff)}(\omega_{\rm wob})\mathscr{J}_z^{\rm (eff)}(\omega_{\rm wob})}}, \tag{3}$$

which is the same form as that in the Bohr-Mottelson model (Eq.(2)) although $\mathscr{J}_y^{\rm (eff)}$ and $\mathscr{J}_z^{\rm (eff)}$ depend on the excitation energy. Note that it is not trivial whether this equation has a collective solution or not. Figure 1 shows the moments of inertia calculated for the 2QP TSD states in ^{168}Hf at a low rotational frequency. This figure clearly indicates that the γ dependence of the calculated moments of inertia resembles that of the irrotational model (Eq.(1)) for $\gamma < 0$, the so-called γ-reversed model,

$$\mathscr{J}_k^{\rm rev} = 4B\beta^2 \sin^2(-\gamma + \frac{2}{3}\pi k), \tag{4}$$

for $0 < \gamma < 40°$, and the inertia of the rigid body,

$$\mathscr{J}_k^{\rm rig} = \mathscr{J}_0 \left(1 - \sqrt{\frac{5}{4\pi}}\beta \cos(\gamma + \frac{2}{3}\pi k)\right), \tag{5}$$

with $\mathscr{J}_0$ the rigid moment of inertia in the spherical limit, for $50° \leq \gamma \leq 60°$. This result can be interpreted as obtained by superimposing the contribution of the aligned QPs on that of the collective rotation. In other words, the above mentioned puzzle has been solved by considering the contribution of the aligned QPs since the calculation results in $\mathscr{J}_x > \mathscr{J}_y^{(\rm eff)}$ in the whole calculated range of γ.

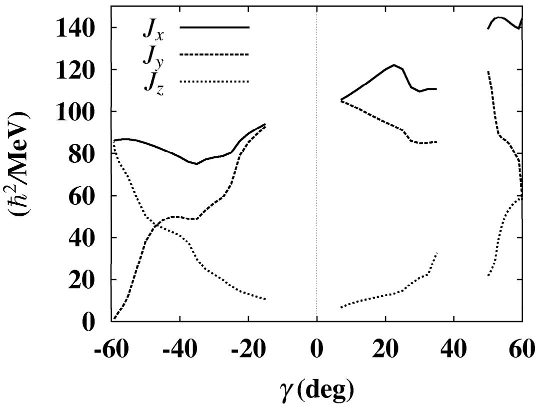

FIGURE 1. Triaxiality dependence of three moments of inertia associated with the wobbling motion in ^{168}Hf, calculated at $\hbar\omega_{\rm rot} = 0.25$ MeV with $\varepsilon_2 = 0.43$ and $\Delta_n = \Delta_p = 0.3$ MeV. (Taken from Ref. [5])

Another evidence that exhibits the necessity of the aligned QP(s) for the existence of the wobbling mode is Fig. 2. These figures present the rotational frequency dependence. Figure 2(a) graphs the excitation energy. This clearly indicates that the wobbling mode emerges when the $(\pi i_{13/2})^2$ aligns as shown in Fig. 2(b).

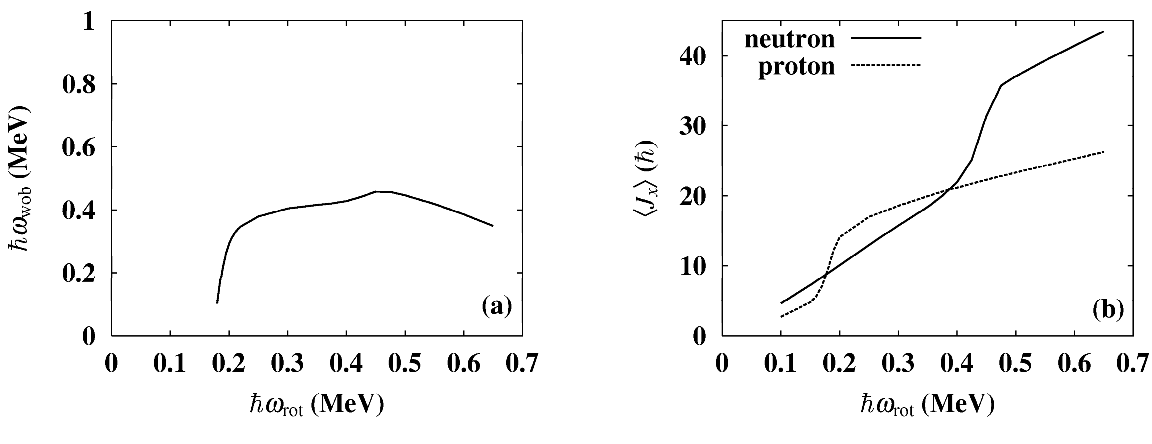

FIGURE 2. Rotational frequency dependence of (a) excitation energy of the wobbling motion and (b) expectation values of angular momenta in the yrast state in ^{168}Hf, calculated with $\varepsilon_2 = 0.43$, $\gamma = 20°$, and $\Delta_n = \Delta_p = 0.3$ MeV. (Taken from Ref. [5])

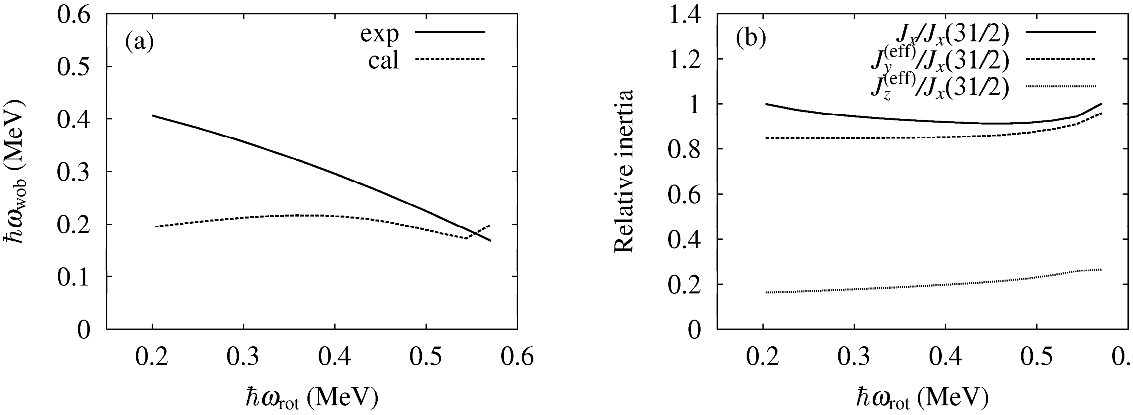

FIGURE 3. (a) Excitation energy and (b) moments of inertia of the TSD2 band in ^{163}Lu as functions of the rotational frequency, calculated with $\varepsilon_2 = 0.43$, $\gamma = 20°$, and $\Delta_n = \Delta_p = 0.3$ MeV. Here the latter were given by normalized to $\mathscr{J}_x(31/2) = 99.2\hbar^2/\mathrm{MeV}$. The proton BC crossing occurs at $\hbar\omega_{\rm rot} \geq 0.55$ MeV in the calculation. Experimental values were calculated from the energy levels in Refs. [6]. (Taken from Ref. [7])

Figure 3(a) shows the observed [6] and calculated excitation energy of the wobbling motion in ^{163}Lu. The most important feature of the observed one is that it is decreasing with respect to the rotational frequency whereas Eq.(2) claims that the excitation energy is proportional to the rotational frequency if moments of inertia are constant. The result of our calculation is not fully decreasing, rather flat, but it approaches the observed one at high rotational frequencies. This is brought about by the decrease of $\mathscr{J}_x - \mathscr{J}_y^{(\rm eff)}$ presented in Fig. 3(b); this effect compensates the increasing tendency due to the direct dependence on the rotational frequency. Another important feature of the data that makes the wobbling character definite is the strength of the interband $B(E2)$ between the wobbling (TSD2) band and the yrast (TSD1) band. Our microscopic calculation accounts for only about 1/2 of the observed strength although the calculated RPA solution is extremely collective.

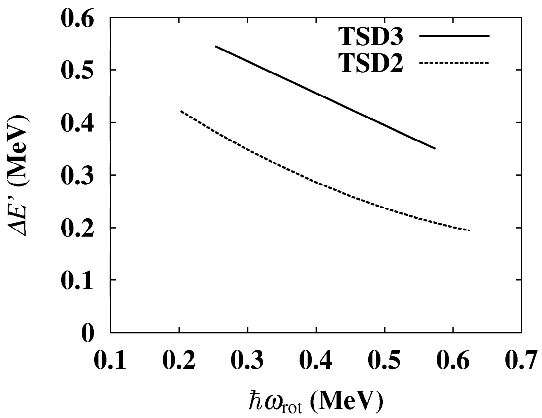

FIGURE 4. Experimental excitation energies of the two-phonon (TSD3) and one-phonon (TSD2) wobbling states relative to the yrast triaxial superdeformed (TSD1) states in ^{163}Lu. Data are taken from Ref. [8]. (Taken from Ref. [9])

Let us point out two open problems of interest: 1) Anharmonicity of the wobbling motion and 2) Limit of softening of the potential energy surface with respect to the wobbling degree of freedom. Reference [8] reported the two-phonon wobbling excitation in ^{163}Lu. Again the strength of the interband $B(E2)$ was decisive for identifying the character of the observed band. The most important characteristic of the data is that the excitation energy is anharmonic as shown in Fig. 4, i.e., the intrinsic excitation energy of the two-phonon band is lower than the twice of that of the one-phonon band. To analyze the anharmonic effects, it would be interesting to go beyond the RPA and apply the self-consistent collective coordinate method [10] to the wobbling motion under consideration. It is expected that the mean field undergoes a “phase transition" in the limit of softening of the potential energy surface with respect to the

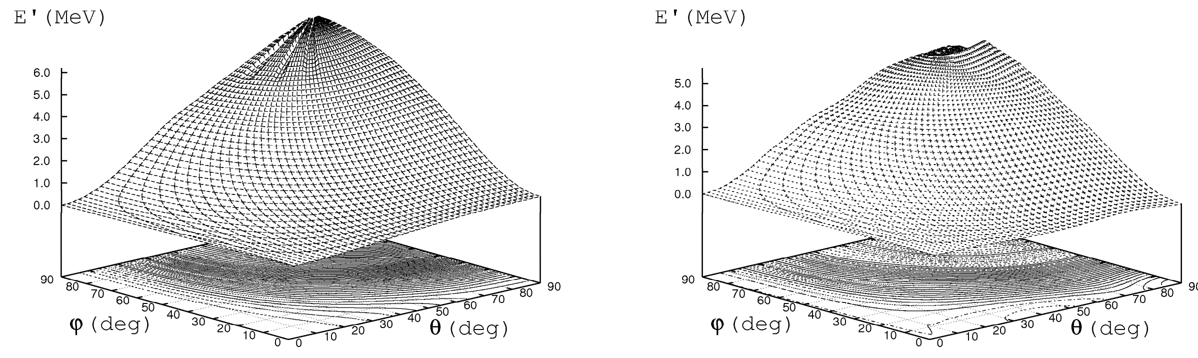

FIGURE 5. Energy surface of the triaxial superdeformed one-quasiparticle configuration in ^{163}Lu (left) and the zero-quasiparticle configuration in ^{162}Yb (right) as a function of tilting angles (θ, φ) calculated at $\hbar\omega_{\rm rot} = 0.5$ MeV with $\varepsilon_2 = 0.43$, $\gamma = 20°$, and $\Delta_n = \Delta_p = 0.3$ MeV. The interval of contours is 100 keV. (Taken from Ref. [9])

wobbling degree of freedom. In the present case, a one-dimensionally (principal axis) rotating mean field undergoes that to a three-dimensionally (tilted axis) rotating one. The signature of this phase transition is such that the excitation energy of the wobbling mode that is excited on top of the one-dimensionally rotating mean field becomes imaginary. The discussion above indicates that this is brought about by $\mathscr{J}_x - \mathscr{J}_y < 0$. In other words, the loss of the alignment to the x axis leads to the phase transition to a tilted axis rotation. In order to see this visually, we calculated the potential surface for the angular frequency vector as a function of tilting angles for the 1QP TSD configuration in ^{163}Lu (Fig. 5 left) and the corresponding 0QP one in ^{162}Yb (right). In these figures polar coordinates are defined with respect to the x axis. Therefore $(\theta, \varphi) = (0,0)$, $(90°, 0)$, and $(90°, 90°)$ correspond to the x, y, and z axes, respectively. In the left figure with an aligned QP, the angular frequency vector sits around the x axis, that is, a principal axis rotation. Whereas, in the right figure with the aligned QP removed, the angular frequency vector is tilted to the $x-y$ plane, that is, a tilted axis rotation.

To summarize, we have discussed some characteristics of the one-phonon and the two-phonon wobbling excitations in triaxial superdeformed nuclei, which are determined by the behavior of the moments of inertia. First we have shown that the wobbling motion in positive γ nuclei emerges thanks to the alignment contribution to the moment of inertia superimposed on the collective contribution. Second we have discussed that the decreasing behavior of the observed excitation energy of the one-phonon wobbling is brought about by the rotational frequency dependence of the dynamical moments of inertia. Finally we have discussed a possible “phase transition" to the tilted axis rotation regime, associated with the instability with respect to the wobbling degree of freedom.

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