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Magnetic Properties of Precession Modes Built on High-K Multi-Quasiparticle States in ^{178}W

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We present an example that shows that the random phase approximation applied to high-K multi-quasiparticle configurations leads to a rotor picture. This is accomplished by calculating excitation energies and magnetic properties of ^{178}W. We also deduce the effective g_R of high-K rotors and compare it with that of low-K rotors.

Rotation is one of typical collective motions in atomic nuclei. Normally, axially symmetric nuclei possess large angular momenta in the form of collective rotation about an axis perpendicular to the symmetry axis. In some cases, in which single particle orbitals with large angular momenta j_i and their projections onto the symmetry axis Ω_i lie in the vicinity of the Fermi surface (realized typically in the $A \sim 180$ region), the nucleus can acquire a large angular momentum through the alignment of multiple quasiparticles (QPs) along the symmetry axis. Sometimes in the latter case yrast states are formed, or even if they are not formed, isomers are often formed owing to the largeness of $K = \sum_i \Omega_i$.

Detailed information regarding high-K configurations can be obtained from their magnetic properties — static magnetic moments and/or g-factors inferred from $B(M1)/B(E2)$ branching ratios of in-band transitions in rotational bands excited from high-K configurations. Data of the latter type are transformed into $|g_K - g_R|/Q_0$ by use of the rotor model.[1)] Then, by employing appropriate values of g_R and Q_0, the extracted g_K is compared with the weighted average of single-j g factors with respect to Ω_i.[2)]

In a different approach, (at least the lower members of) rotational bands excited from high-K configurations can be described as multiple excitations of the precession phonons in the language of the random phase approximation (RPA).[3),4)] Thus, by calculating the wave function of the one phonon state, $B(M1 : I = K + 1 \to K)$ can be obtained and transformed into the effective (RPA) $(g_K - g_R)$; its sign can be determined from the calculated $E2/M1$ mixing ratio. The magnetic moment $\langle\mu\rangle$, and accordingly the g factor, $g = \sqrt{\frac{4\pi}{3}}\langle\mu\rangle/(\langle J\rangle\mu_N)$, of high-$K$ configurations can be calculated at the mean field level. Because this g essentially coincides with g_K, we can deduce the value of g_R for a high-K rotor by combining the RPA $(g_K - g_R)$ and

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g.

The purpose of this paper is twofold: First, by applying the above method to ^{178}W, for which the richest experimental information[5)–7)] is available, we confirm that the RPA yields a rotor picture via the excitation energies and magnetic properties, and second, we deduce g_R for high-K rotors.

We begin with the following one-body Hamiltonian:

$$\begin{aligned} h' &= h - \hbar\omega_{\rm rot} J_x, \\ h &= h_{\rm Nil} - \Delta_\tau (P_\tau^\dagger + P_\tau) - \lambda_\tau N_\tau, \\ h_{\rm Nil} &= \frac{\boldsymbol{p}^2}{2M} + \frac{1}{2} M(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) + v_{ls} \boldsymbol{l}\cdot\boldsymbol{s} + v_{ll}(\boldsymbol{l}^2 - \langle \boldsymbol{l}^2 \rangle_{N_{\rm osc}}). \end{aligned} \tag{1}$$

Here $\tau = 1$ and 2 stand for the neutron and the proton, respectively, and the chemical potentials λ_τ are determined so as to give the correct average particle numbers, $\langle N_\tau \rangle$. The oscillator frequencies are related to the quadrupole deformation parameters ϵ_2 and γ in the usual way. (We adopt the so-called Lund convention.) The orbital angular momentum $\boldsymbol{l}$ is defined in the singly-stretched coordinates $x'_k = \sqrt{\frac{\omega_k}{\omega_0}} x_k$, with $k = 1$, 2 and 3 denoting x, y and z, respectively, and the corresponding momenta. Nuclear states with QP excitations, i.e., alignments along the x axis, are obtained by exchanging the QP energy and wave functions as

$$(-e'_\mu, \boldsymbol{V}_\mu, \boldsymbol{U}_\mu) \to (e'_{\bar\mu}, \boldsymbol{U}_{\bar\mu}, \boldsymbol{V}_{\bar\mu}), \tag{2}$$

where $\bar\mu$ denotes the signature partner of μ.

We apply the RPA to the residual pairing plus doubly-stretched quadrupole-quadrupole ($Q'' \cdot Q''$) interaction between QPs. Because we are interested in the precession mode that has a definite signature quantum number, $\alpha = 1$, only two of the five components of the $Q'' \cdot Q''$ interaction are relevant. These are the $K = \pm 1$ components. Note that we refer to the symmetry axis, with respect to which the K quantum number is defined, as the x axis throughout this paper; that is, we consider the $\gamma = -120^\circ$ case. These components of the interaction are related to the restoration of spherical symmetry. Requiring the decoupling of this symmetry mode (the Nambu-Goldstone mode), $J_\pm = J_y \pm i J_z$, the strength of the interaction is determined. Then, utilizing the identities given in Table III of Ref. 8),[*)] the RPA equation of motion can be cast into[9)] the following form, which we use in the actual calculation, instead of the original equation in terms of Q'' operators:

$$(\omega^2 - \omega_{\rm rot}^2) \begin{vmatrix} A(\omega) & C(\omega) \\ B(\omega) & D(\omega) \end{vmatrix} = 0, \tag{3}$$

where

$$\begin{aligned} A(\omega) &= \omega \mathcal{J}_y(\omega) - \omega_{\rm rot} \mathcal{J}_{yz}(\omega), \\ B(\omega) &= \omega_{\rm rot} \left(\mathcal{J}_y(\omega) - \mathcal{J}_x\right) - \omega \mathcal{J}_{yz}(\omega), \\ C(\omega) &= \omega_{\rm rot} \left(\mathcal{J}_z(\omega) - \mathcal{J}_x\right) - \omega \mathcal{J}_{yz}(\omega), \\ D(\omega) &= \omega \mathcal{J}_z(\omega) - \omega_{\rm rot} \mathcal{J}_{yz}(\omega), \end{aligned} \tag{4}$$

[*)] Strictly speaking, these identities have very small errors introduced by the use of singly-stretched (rather than unstretched) $\boldsymbol{l}\cdot\boldsymbol{s}$ and $\boldsymbol{l}^2$ potentials.

with

$$\begin{aligned}
\mathcal{J}_x &= \hbar\langle J_x\rangle/\omega_{\rm rot},\\
\mathcal{J}_y(\omega) &= \sum_{\mu<\nu}^{(\alpha=\pm1/2)} \frac{2E_{\mu\nu}\left(iJ_y(\mu\nu)\right)^2}{E_{\mu\nu}^2-(\hbar\omega)^2},\\
\mathcal{J}_z(\omega) &= \sum_{\mu<\nu}^{(\alpha=\pm1/2)} \frac{2E_{\mu\nu}\left(J_z(\mu\nu)\right)^2}{E_{\mu\nu}^2-(\hbar\omega)^2},\\
\mathcal{J}_{yz}(\omega) &= \sum_{\mu<\nu}^{(\alpha=\pm1/2)} \frac{2\hbar\omega iJ_y(\mu\nu)J_z(\mu\nu)}{E_{\mu\nu}^2-(\hbar\omega)^2}.
\end{aligned} \tag{5}$$

Here, we adopt the convention in which the matrix elements of J_y and μ_y (below) are purely imaginary. Then, the non-spurious part of Eq. (3), $A(\omega)D(\omega)-B(\omega)C(\omega)=0$, can be rewritten as

$$\left[\omega\mathcal{J}_+^{(\rm eff)}(\omega)-\omega_{\rm rot}\left(\mathcal{J}_x-\mathcal{J}_+^{(\rm eff)}(\omega)\right)\right]\left[\omega\mathcal{J}_-^{(\rm eff)}(\omega)+\omega_{\rm rot}\left(\mathcal{J}_x-\mathcal{J}_-^{(\rm eff)}(\omega)\right)\right]=0, \tag{6}$$

where the subscripts + and − refer to the $\Delta K=+1$ and -1 modes, respectively, and we have

$$\begin{aligned}
\mathcal{J}_\pm^{(\rm eff)}(\omega) &= \mathcal{J}_\perp(\omega)\mp\mathcal{J}_{yz}(\omega),\\
\mathcal{J}_\perp(\omega) &= \mathcal{J}_y(\omega)=\mathcal{J}_z(\omega).
\end{aligned} \tag{7}$$

For $\Delta K=+1$ excitations, corresponding to the precession modes, the excitation energy in the laboratory frame is given by

$$\hbar\omega+\hbar\omega_{\rm rot}=\hbar\omega_{\rm rot}\frac{\mathcal{J}_x}{\mathcal{J}_+^{(\rm eff)}(\omega)}=\hbar^2\frac{\langle J_x\rangle}{\mathcal{J}_+^{(\rm eff)}(\omega)}, \tag{8}$$

which is independent of $\omega_{\rm rot}$. Because the excitation energy of the first rotational state for the high-K configuration in the rotor model is given by

$$E_{I=K+1}-E_{I=K}=\frac{\hbar^2}{\mathcal{J}}\left(K+1\right), \tag{9}$$

which is derived from

$$E_I=\frac{\hbar^2}{2\mathcal{J}}\left(I(I+1)-K^2\right), \tag{10}$$

Eq. (8) ($\langle J_x\rangle=K$ in the cases $\gamma=-120^\circ$ and 60°) and Eq. (9) correspond to each other well for $K\gg1$. In other words, $\mathcal{J}_+^{(\rm eff)}(\omega)$ in our RPA formalism and $\mathcal{J}$ in the axially symmetric rotor model correspond to each other.

Marshalek derived an expression for multipole transition rates, which is valid for $I\gg1$, in terms of the RPA wave function.[10] In the $M1$ case, it reads

$$\begin{aligned}
B(M1:I\to I-1) &= \frac{1}{2}\langle[i\mu_y+\mu_z,X_n^\dagger]\rangle^2,\\
\mu_{y(z)} &= \sqrt{\frac{3}{4\pi}}\mu_N\left(g_l l_{y(z)}+g_s^{(\rm eff)}s_{y(z)}\right),
\end{aligned} \tag{11}$$

for the n-th phonon state. Hereafter we concentrate on the precession phonon. By equating this with the expression in the rotor model,[1)]

$$B(M1 : I = K + 1 \to K) = \frac{3}{4\pi}\mu_N^2 \left(g_K - g_R\right)^2 K^2 \langle IK10|I - 1K\rangle^2, \qquad (12)$$

we obtain the RPA $|g_K - g_R|$. Its sign is determined by that of the calculated $E2/M1$ mixing ratio.

Calculations are performed for all the high-K (4, 6, 8 and 10QP) configurations that possess rotational bands: $K^\pi = 13^-$, 14^+, 15^+, 18^-, 21^-, 22^-, 25^+, 28^-, 29^+, 30^+ and 34^+. The configuration for $K^\pi = 13^-$, 14^+ and 15^+ is $2\nu 2\pi$, that for 18^- is $2\nu 4\pi$, that for 21^- and 22^- is $4\nu 2\pi$, that for 25^+, 28^- and 30^+ is $4\nu 4\pi$, and that for 29^+ and 34^+ is $6\nu 4\pi$.[6),7)] The model space is $N_{\rm osc} = 3$ – 7 for neutrons and $N_{\rm osc} = 2$ – 6 for protons. The strengths of the $\boldsymbol{l}\cdot\boldsymbol{s}$ and $\boldsymbol{l}^2$ potentials are taken from Ref. 11). The pairing gaps are assumed to be 0.5 MeV for 2QP and 0.01 MeV for 4QP and 6QP configurations, both for neutrons and protons. The quadrupole deformation is chosen to be $\epsilon_2 = 0.235$, which reproduces, after taking a rough average, the value $Q_0 = 7.0$ eb, which was assumed in experimental analyses.[6),7)] For the spin g factor, $g_s^{(\rm eff)} = 0.7 g_s^{(\rm free)}$ is adopted, as usual. Hence, the choice of parameter values in this work is semi-quantitative; we checked the robustness of the results with respect to the variations of these parameter values. In the cases symmetric with respect to the x axis considered here, the results do not depend on $\omega_{\rm rot}$, while the actual calculations were performed with $\hbar\omega_{\rm rot} = 0.001$ MeV.

Figure 1 presents the calculated and observed relative excitation energies for the first rotational band members, $E_{I=K+1} - E_{I=K}$. We find that our RPA calculation reproduces their gross features well, but there are deviations at $K^\pi = 18^-$, 25^+, 28^- and 29^+, which include the $\pi h_{9/2}$ orbital. Note that here small calculated energies correspond to large moments of inertia [see Eq. (8)], and their size is correlated with that of the calculated Q_0. The large size of Q_0 indicates the shape polarization effect of this high-j orbital in the prolate direction. (With regard to the effect of the $\pi h_{9/2}$ orbital on the moment of inertia, see also Refs. 12) and 13).)

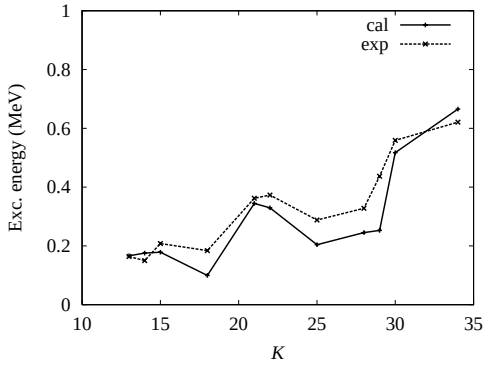

Fig. 1. Calculated and experimental excitation energies of the first rotational band members, $E_{I=K+1} - E_{I=K}$. The data are taken from Refs. 6) and 7).

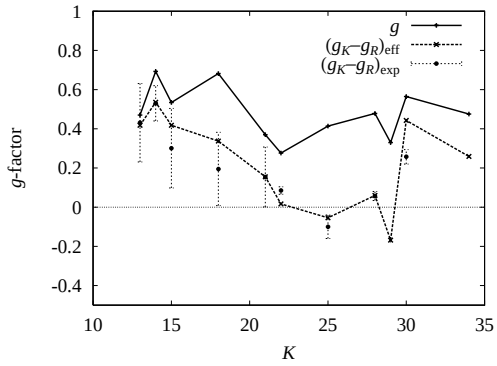

Fig. 2. Calculated intrinsic g of high-K configurations (solid curve) and calculated (dashed curve) and experimental (points with error bars) $(g_K - g_R)$. The data are taken from Refs. 6) and 7).

In Fig. 2 we compare the RPA $(g_K - g_R)$ extracted from the calculated $B(M1; I = K + 1 \rightarrow K)$ and the observed one extracted from the branching ratios of the lowest available transitions in the respective rotational bands, assuming $Q_0 = 7.0$ eb. They agree with each other well. In this figure, the calculated intrinsic $g = \sqrt{\frac{4\pi}{3}}\langle\mu_x\rangle/(\langle J_x\rangle\mu_N)$ of high-K configurations are also shown. The two calculated curves are roughly parallel. According to the relation[1)]

$$g_R = g - (g_K - g_R)\frac{K^2}{I(I+1)}, \tag{13}$$

with $I = K + 1$, g is almost equal to g_K. Consequently, the difference between the two curves essentially corresponds to the effective g_R for high-K cases. Thus, this correlation suggests the possibility of deducing the effective g_R of the high-K configurations under consideration by substituting the RPA $(g_K - g_R)$ into Eq. (13). Its average value is approximately 0.29, as seen from Fig. 3. This value may represent a rough measure of a property of high-K rotors. Moreover, an interesting feature is that there are significant variations, and the variations for the configurations including the $\pi h_{9/2}$ orbital are larger than those for the others. In order to see this more clearly, in Fig. 3 we compare the above-deduced values with those calculated using the approximate relation[14)]

$$g_R = \frac{\mathcal{J}_\pi}{\mathcal{J}_\nu + \mathcal{J}_\pi}, \tag{14}$$

where the neutron and proton parts of the effective inertia, the upper sign in Eq. (7), are substituted into $\mathcal{J}_\nu$ and $\mathcal{J}_\pi$. It is clear that the values of g_R deduced from Eqs. (13) and (14) exhibit similar dependence on K, although those from Eq. (14) are much larger for the configurations in which the $\pi h_{9/2}$ orbital is occupied. This is due to the fact that the contribution to the moment of inertia from the $\pi h_{9/2}$ orbital is large and overestimated in the calculation, as mentioned above in the case of excitation energies.

Finally, we compare g_R for high-K rotors above and that of the ground state band. We calculated g of the 2^+, 4^+ and 6^+ states (which in the zeroth-order

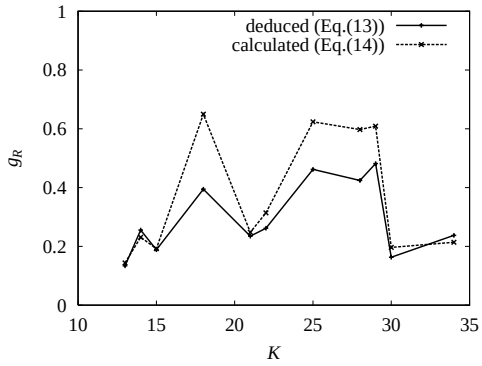

Fig. 3. g_R of high-K rotors deduced using Eq. (13) (solid curve) and calculated using Eq. (14) (dashed curve).

approximation plays the role of g_R for the nearby $K \neq 0$ configurations) using $\hbar\omega_{\rm rot} = 0.053$, 0.119 and 0.176 MeV, respectively, with $\epsilon_2 = 0.235$, $\gamma = 0$ and the odd-even mass differences $\Delta_n = 0.883$ MeV and $\Delta_p = 1.026$ MeV as the pairing gaps. The results are $g = 0.218$, 0.216 and 0.214, respectively, which are almost equal to the average values for the configurations that do not include the $\pi h_{9/2}$ orbital. This indicates that high-K and low-K rotors are similar, unless the shape driving $\pi h_{9/2}$ orbital is included.

To summarize, we have numerically verified that the random phase approximation applied to high-K multi-quasiparticle configurations leads to a rotor picture, as previously demonstrated using $E2$ properties by Andersson et al.,[3] by calculating excitation energies and $M1$ properties. Next, we deduced the effective g_R for high-K rotors and compared its values with those of the low-K rotor near the ground state. A more detailed investigation is in progress.

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