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SIGNATURE-DEPENDENT EFFECTS OF GAMMA VIBRATION ON E2 TRANSITIONS IN ROTATING ODD-*A* NUCLEI

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Abstract: The origin of the signature dependence of E2-transition matrix elements is studied by analysing the microscopic structures of the quasiparticle-vibration-coupling wave functions. We show that the phase rule of signature dependence stemming from the gamma-vibrational contributions can be related to the signature splitting of quasiparticle energies. We also discuss the nucleon-number dependence of vibrational effects and show that the results of various models can be understood consistently.

1. Introduction

Recent experimental progress has made it possible to measure not only energy spectra but also electromagnetic transition probabilities between high-spin states in the near-yrast region. These quantities give us information about the static and the dynamic nuclear deformations at the high-spin states. The effects of the static triaxial deformation on electric quadrupole transitions in high-j unique-parity bands in odd-A nuclei were studied by Hamamoto and Mottelson [1]). They showed an axially asymmetric nuclear shape gives rise to the signature dependence of the E2-transition matrix elements. Later, Ikeda [2]) showed that the gamma vibrations in axially symmetric nuclei also bring about the signature dependence. Both calculations reproduced the staggering in the $B(\mathrm{E2}, \Delta I=1)$ values, which is in the same phase with the experimental data for ^{157}Ho [ref. [3])] where the odd-quasiproton lies in the mid $\pi h_{11/2}$ shell region. On the other hand, Onishi *et al.* [4]) made a calculation in which both the static triaxial deformation and the fluctuation around it are taken into account, and they obtained the signature dependence of $B(\mathrm{E2}, \Delta I=1)$ whose phase is opposite to the experimental data in ref. [3]). In these works, the particle-rotor model was used and it was found numerically that the $K=2$ component of the quadrupole moments brings about the signature dependence of the E2-transition rates.

In the preceding paper [5]) we proposed a new microscopic model which can take into account both the static and the dynamic triaxial deformations and analyzed available experimental data for electromagnetic transition rates in rotating odd-A

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nuclei. The effective intrinsic operator for non-stretched E2 transitions in this microscopic model [5]) has three parts: the rotational, the vibrational and the odd-quasiparticle parts. As was shown in refs. [5,6]), we can see directly from the operator-form of the rotational part, which contains the static quadrupole moments Q_0 and Q_2, how the static triaxial deformation affects the $B(\mathrm{E2}, \Delta I=1)$ values. On the other hand, the signature dependence of the matrix elements associated with the vibrational part is not easy to understand, because the properties of these matrix elements are determined by the detailed properties of the quasiparticle-vibration-coupling wave functions.

In the present paper, we study the microscopic mechanism of the signature dependence of $B(\mathrm{E2}, \Delta I=1)$ stemming from the vibrational contributions in high-j, unique parity bands in odd-A nuclei. We do this by analyzing the structures of the quasiparticle-vibration-coupling wave functions. The nucleon-number dependence of the signature-dependent effects is studied both analytically and numerically. We show that the phase rule of the signature dependence is related to the signature splitting of quasiparticle energies.

We give a brief review of our formulation in sect. 2. In sect. 3 the origin of the signature dependence of the E2-transition matrix elements is studied. The nucleon-number dependence is discussed in sect. 4. Concluding remarks are given in sect. 5.

2. Structures of the E2 matrix elements

The quasiparticle-vibration-coupling hamiltonian can be derived from the pairing plus doubly stretched quadrupole interaction in rotating frame, and takes the following form*:

$$\begin{aligned}\hat{H}_{\mathrm{coupl}}(\gamma) = {\sum_{\mu\nu}}'' \Lambda_\gamma^{(+)}(\mu\nu)(X^\dagger_{\gamma(+)} a^\dagger_\mu a_\nu + X_{\gamma(+)} a^\dagger_\nu a_\mu) \\ + \sum_{\mu\bar{\nu}} \Lambda_\gamma^{(-)}(\mu\bar{\nu})(X^\dagger_{\gamma(-)} a^\dagger_\mu a_{\bar{\nu}} + X_{\gamma(-)} a^\dagger_{\bar{\nu}} a_\mu) \\ + \sum_{\mu\bar{\nu}} \Lambda_\gamma^{(-)}(\bar{\nu}\mu)(X^\dagger_{\gamma(-)} a^\dagger_{\bar{\nu}} a_\mu + X_{\gamma(-)} a^\dagger_\mu a_{\bar{\nu}}) .\end{aligned} \tag{2.1}$$

The coupling vertices with gamma-vibrational phonons $X^\dagger_{\gamma(\pm)}$ are classified by the signature quantum number $r=\pm 1$, and are given by**

$$\begin{aligned}\Lambda_\gamma^{(+)}(\mu\nu) &= -\sum_{K=0,1,2} \kappa_K^{(+)} \tilde{T}_K^{(+)} \tilde{Q}_K^{(+)}(\mu\nu) , \\ \Lambda_\gamma^{(-)}(\mu\bar{\nu}) &= -\sum_{K=1,2} \kappa_K^{(-)} \tilde{T}_K^{(-)} \tilde{Q}_K^{(-)}(\mu\bar{\nu}) , \\ \Lambda_\gamma^{(-)}(\bar{\nu}\mu) &= -\sum_{K=1,2} \kappa_K^{(-)} \tilde{T}_K^{(-)} \tilde{Q}_K^{(-)}(\bar{\nu}\mu) ,\end{aligned} \tag{2.2}$$

* The double prime attached to $\sum$ denotes that when the component $(\mu\nu)$ is summed its signature partner $(\bar{\mu}\bar{\nu})$ should also be summed.

** The contributions from the residual pairing interaction in the $r=+1$ sector are omitted in this expression for simplicity.

where

$$\tilde{T}_K^{(\pm)} = [\tilde{Q}_K^{(\pm)}, X^\dagger_{\gamma(\pm)}]_{\text{RPA}} , \tag{2.3}$$

with $\tilde{Q}_K^{(\pm)}$s denoting the doubly stretched quadrupole operators [7]. Diagonalization of $\hat{H}_{\text{coupl}}(\gamma)$ gives the intrinsic wave functions of the following form:

$$\begin{aligned}|\chi_n(\omega_{\text{rot}})\rangle &= \sum_\mu \psi_n^{(1)}(\mu) a^\dagger_\mu |\phi\rangle + \sum_\mu \psi_n^{(3)}(\mu\gamma) a^\dagger_\mu X^\dagger_{\gamma(+)}|\phi\rangle \\ &+ \sum_{\bar{\mu}} \psi_n^{(3)}(\bar{\mu}\bar{\gamma}) a^\dagger_{\bar{\mu}} X^\dagger_{\gamma(-)}|\phi\rangle + \cdots \qquad (\text{for the } r=-i \text{ sector}) ,\end{aligned} \tag{2.4}$$

where $|\phi\rangle$ is a triaxially deformed reference state in even–even nuclei.

The effective operator in the principal axis (PA) frame for the E2($\Delta I = 1$) transitions is written as

$$\frac{1}{i}\hat{Q}^{(\text{PA})}_{2-1} = \left\{ -\sqrt{\tfrac{3}{2}} Q_0 \frac{\hat{J}_z^{(\text{qp})}}{I_0} + Q_2\left(2\frac{i\hat{J}_y^{(\text{qp})}}{I_0} + \frac{\hat{J}_z^{(\text{qp})}}{I_0}\right)\right\} + \frac{1}{i}\hat{Q}^{(\text{vib})}_{2-1} + \frac{1}{i}\hat{Q}^{(\text{qp})}_{2-1} , \tag{2.5}$$

where operators $\hat{Q}_{2-1}$ and expectation values Q_K ($K = 0, 2$) are quantized along the x-axis and the z-axis, respectively, with

$$\frac{1}{i}\hat{Q}^{(\text{vib})}_{2-1} = \sum_n \left\{ \left[X_n, \frac{1}{i}\hat{Q}_{2-1}\right]_{\text{RPA}} X^\dagger_n - \left[X^\dagger_n, \frac{1}{i}\hat{Q}_{2-1}\right]_{\text{RPA}} X_n \right\} . \tag{2.6}$$

The matrix elements for the transitions from the $r = +i$ to the $r = -i$ states are given by

$$\begin{aligned}
&\langle \chi_n(\omega_{\text{rot}})| \frac{1}{i}\hat{Q}^{(\text{PA})}_{2-1} |\chi_{\bar{n}'}(\omega_{\text{rot}})\rangle \\
&= \sum_{\mu\bar{\nu}} (\psi_n^{(1)}(\mu)\psi_{\bar{n}'}^{(1)}(\bar{\nu}) + \psi_n^{(3)}(\mu\gamma)\psi_{\bar{n}'}^{(3)}(\bar{\nu}\gamma)) \\
&\quad\times\left\{ -\sqrt{\tfrac{3}{2}} Q_0 \frac{J_z(\mu\bar{\nu})}{I_0} + Q_2\left(2\frac{iJ_y(\mu\bar{\nu})}{I_0} + \frac{J_z(\mu\bar{\nu})}{I_0}\right) + \sqrt{\tfrac{1}{2}}(Q_1^{(-)}(\mu\bar{\nu}) - Q_2^{(-)}(\mu\bar{\nu}))\right\} \\
&\quad+ \sum_{\mu\bar{\nu}} \psi_n^{(3)}(\bar{\nu}\bar{\gamma})\psi_{\bar{n}'}^{(3)}(\bar{\mu}\bar{\gamma}) \\
&\quad\times\left\{ -\sqrt{\tfrac{3}{2}} Q_0 \frac{J_z(\mu\bar{\nu})}{I_0} + Q_2\left(-2\frac{iJ_y(\mu\bar{\nu})}{I_0} + \frac{J_z(\mu\bar{\nu})}{I_0}\right) + \sqrt{\tfrac{1}{2}}(Q_1^{(-)}(\mu\bar{\nu}) + Q_2^{(-)}(\mu\bar{\nu}))\right\} \\
&\quad+ \left(\sum_\mu \psi_n^{(1)}(\mu)\psi_{\bar{n}'}^{(3)}(\mu\bar{\gamma})\right)\sqrt{\tfrac{1}{2}}(T_1^{(-)} - T_2^{(-)}) \\
&\quad+ \left(\sum_{\bar{\nu}} \psi_n^{(3)}(\bar{\nu}\bar{\gamma})\psi_{\bar{n}'}^{(1)}(\bar{\nu})\right)\sqrt{\tfrac{1}{2}}(T_1^{(-)} + T_2^{(-)}) ,
\end{aligned} \tag{2.7}$$

where we used the relation [7]

$$\frac{1}{i}\hat{Q}_{2-1} = \sqrt{\tfrac{1}{2}}(\hat{Q}_1^{(-)} - \hat{Q}_2^{(-)}) , \tag{2.8}$$

when evaluating the last two terms in eq. (2.5). The vibrational contributions are given by the last two lines in eq. (2.7) including the RPA transition amplitudes $T_K^{(-)}$ ($K=1,2$) associated with the gamma-vibrational phonon. Matrix elements for the transitions from the $r=-i$ to the $r=+i$ states can be obtained in the same way. We see from these expressions that the anti-hermitian parts that involve the operator $i\hat{J}_y$ and the $K=2$ contribution $\hat{Q}_2^{(-)}$ give rise to the signature dependence in all the three parts in eq. (2.5).

The phase rule of the signature dependence stemming from the rotational part can be obtained by the approximate relation [8,5,6])

$$i\hat{J}_y^{(\mathrm{qp})} \simeq (-1)^{I_i-j} \frac{|\Delta E|}{\hbar\omega_{\mathrm{rot}}} \hat{J}_z^{(\mathrm{qp})}, \tag{2.9}$$

where I_i and j denote the angular momenta of the initial state and the odd-quasiparticle in the high-j, unique parity orbit, respectively, and $\hbar\omega_{\mathrm{rot}}$ and ΔE are the rotational frequency and the signature splitting of the quasiparticle energies. Using this relation we can rewrite the rotational part of eq. (2.5) as

$$\left\{-\sqrt{\tfrac{3}{2}}Q_0 + Q_2\left(2(-1)^{I_i-j}\frac{|\Delta E|}{\hbar\omega_{\mathrm{rot}}}+1\right)\right\}\frac{\hat{J}_z^{(\mathrm{qp})}}{I_0}, \tag{2.10}$$

in a good approximation. This expression shows that, if the vibrational and the odd-quasiparticle contributions are neglected, $B(\mathrm{E2}, \mathrm{u}\to\mathrm{f})$ is larger than $B(\mathrm{E2}, \mathrm{f}\to\mathrm{u})$ when Q_0 and Q_2 have the same sign. Henceforth we use the notations f and u for the favoured state ($I-j=$ even) and the unfavoured state ($I-j=$ odd), respectively. Hamamoto obtained [8]) an expression similar to eq. (2.10). But her expression applies only for the case of $j=\frac{1}{2}$.

3. Origin of the signature dependence of the E2-transition matrix elements associated with the vibrational part

As was pointed out in sect. 2, the signature dependence of $B(\mathrm{E2}, \Delta I=1)$ associated with the vibrational contributions comes from the difference of the relative phase between the main term involving Q_0 and the terms involving $T_2^{(-)}$. This phase difference is a consequence of the anti-hermitian property of $\hat{Q}_2^{(-)}$. While the phase rule is determined by the properties of the quasiparticle-vibration-coupling wave functions.

The vertices $\Lambda_\gamma^{(-)}(\mu\bar{\nu})$ in eq. (2.2) give the coupling strengths between the one-quasiparticle state $|r=+i\rangle$ and the one-quasiparticle-one-gamma-vibrational state $X_{\gamma(-)}^\dagger|r=-i\rangle$, while $\Lambda_\gamma^{(-)}(\bar{\nu}\mu)$ give those between $|r=-i\rangle$ and $X_{\gamma(-)}^\dagger|r=+i\rangle$. In other words, when $r=+i$ is the favoured signature $|\alpha; \alpha\in \mathrm{f}\rangle$ and $X_{\gamma(-)}^\dagger|\beta; \beta\in \mathrm{u}\rangle$ are coupled with the strengths $\Lambda_\gamma^{(-)}(\mu\bar{\nu})$ while $|\alpha; \alpha\in \mathrm{u}\rangle$ and $X_{\gamma(-)}^\dagger|\beta; \beta\in \mathrm{f}\rangle$ are coupled with the strengths $\Lambda_\gamma^{(-)}(\bar{\nu}\mu)$. When $r=-i$ is the favoured signature, the roles of

$\Lambda_\gamma^{(-)}(\mu\bar{\nu})$ and $\Lambda_\gamma^{(-)}(\bar{\nu}\mu)$ are exchanged. Using the symmetry property [7])

$$\tilde{Q}_K^{(-)}(\mu\bar{\nu}) = -(-1)^K \tilde{Q}_K^{(-)}(\bar{\nu}\mu)\,, \tag{3.1}$$

we can rewrite $\Lambda_\gamma^{(-)}(\mu\bar{\nu})$ and $\Lambda_\gamma^{(-)}(\bar{\nu}\mu)$ as

$$\begin{aligned}\Lambda_\gamma^{(-)}(\gamma\mathrm{f},\mathrm{u}) &= -(\kappa_1^{(-)}\tilde{T}_1^{(-)}\langle\mathrm{f}|\tilde{Q}_1^{(-)}|\mathrm{u}\rangle + \kappa_2^{(-)}\tilde{T}_2^{(-)}\langle\mathrm{f}|\tilde{Q}_2^{(-)}|\mathrm{u}\rangle)\,,\\ \Lambda_\gamma^{(-)}(\gamma\mathrm{u},\mathrm{f}) &= -(\kappa_1^{(-)}\tilde{T}_1^{(-)}\langle\mathrm{f}|\tilde{Q}_1^{(-)}|\mathrm{u}\rangle - \kappa_2^{(-)}\tilde{T}_2^{(-)}\langle\mathrm{f}|\tilde{Q}_2^{(-)}|\mathrm{u}\rangle)\,.\end{aligned} \tag{3.2}$$

Using these expressions the wave functions (2.4) for yrast states are reduced, in the first-order perturbation theory, to

$$\begin{aligned}|\mathrm{u}\rangle\rangle &= a_\mathrm{u}^\dagger|\phi\rangle + \delta_\mathrm{u} a_\mathrm{f}^\dagger X_{\gamma(-)}^\dagger|\phi\rangle\,,\\ |\mathrm{f}\rangle\rangle &= a_\mathrm{f}^\dagger|\phi\rangle + \delta_\mathrm{f} a_\mathrm{u}^\dagger X_{\gamma(-)}^\dagger|\phi\rangle\,,\end{aligned} \tag{3.3}$$

with

$$\begin{aligned}\delta_\mathrm{u} &= \frac{\Lambda_\gamma^{(-)}(\gamma\mathrm{f},\mathrm{u})}{\Delta E - \hbar\omega_{\gamma(-)}}\,,\\ \delta_\mathrm{f} &= \frac{\Lambda_\gamma^{(-)}(\gamma\mathrm{u},\mathrm{f})}{-\Delta E - \hbar\omega_{\gamma(-)}}\,,\end{aligned} \tag{3.4}$$

where ΔE is defined by $\Delta E = E_\mathrm{u} - E_\mathrm{f}$.

The matrix element (2.7) and its signature partner can be written as

$$\left.\begin{matrix}(\mathrm{f}\to\mathrm{u})\\(\mathrm{u}\to\mathrm{f})\end{matrix}\right\} = -\sqrt{\tfrac{3}{2}}Q_0\frac{\langle\mathrm{f}|\hat{J}_z|\mathrm{u}\rangle}{I_0} + Q_2\left(\mp 2\frac{\langle\mathrm{f}|i\hat{J}_y|\mathrm{u}\rangle}{I_0} + \frac{\langle\mathrm{f}|\hat{J}_z|\mathrm{u}\rangle}{I_0}\right)$$
$$+\sqrt{\tfrac{1}{2}}(\langle\mathrm{f}|\hat{Q}_1^{(-)}|\mathrm{u}\rangle \pm \langle\mathrm{f}|\hat{Q}_2^{(-)}|\mathrm{u}\rangle) + \sqrt{\tfrac{1}{2}}((\delta_\mathrm{u}+\delta_\mathrm{f})T_1^{(-)} \pm (\delta_\mathrm{u}-\delta_\mathrm{f})T_2^{(-)})\,. \tag{3.5}$$

With the help of eqs. (3.4), (3.2) and (2.3), the linear combinations of perturbation amplitudes in eq. (3.5) can be written as

$$\begin{aligned}\delta_\mathrm{u} + \delta_\mathrm{f} &\simeq \frac{2}{\hbar\omega_{\gamma(-)}}\chi_1^{(-)}T_1^{(-)}\langle\mathrm{f}|\hat{Q}_1^{(-)}|\mathrm{u}\rangle\,,\\ \delta_\mathrm{u} - \delta_\mathrm{f} &\simeq \frac{2}{\hbar\omega_{\gamma(-)}}\chi_2^{(-)}T_2^{(-)}\langle\mathrm{f}|\hat{Q}_2^{(-)}|\mathrm{u}\rangle\,,\end{aligned} \tag{3.6}$$

when $\Delta E \ll \hbar\omega_{\gamma(-)}$ holds. We used the relations between the doubly stretched quantities and non-stretched ones:

$$\tilde{Q}_1^{(-)} = \frac{\omega_z\omega_x}{\omega_0^2}\hat{Q}_1^{(-)}\,, \qquad \tilde{Q}_2^{(-)} = \frac{\omega_x\omega_y}{\omega_0^2}\hat{Q}_2^{(-)}\,, \tag{3.7}$$

and

$$\chi_1^{(-)} = \left(\frac{\omega_z\omega_x}{\omega_0^2}\right)^2\kappa_1^{(-)}\,, \qquad \chi_2^{(-)} = \left(\frac{\omega_x\omega_y}{\omega_0^2}\right)^2\kappa_2^{(-)}\,, \tag{3.8}$$

in the derivations of eq. (3.6). Expressions (3.5) and (3.6) show that the gamma-vibrational contributions bring about the signature dependence such that $B(\mathrm{E2}, \mathrm{f}\to\mathrm{u})$ is larger than $B(\mathrm{E2}, \mathrm{u}\to\mathrm{f})$ when the sign of $\langle \mathrm{f}|\hat{J}_z|\mathrm{u}\rangle$ is opposite to that of $\langle \mathrm{f}|\hat{Q}_2^{(-)}|\mathrm{u}\rangle$ and Q_0 is positive. Note that the odd-quasiparticle contribution also gives rise to the same signature dependence. But its magnitude is much smaller than that of the vibrational contributions discussed above.

The quadrupole operators with $r=-1$ in a single-j shell-model are represented by replacing the coordinate $\boldsymbol{x}$ by the angular momentum $\boldsymbol{J}$ [ref. [9]] as follows:

$$\hat{Q}_1^{(-)} = -2\sqrt{3}c_0\tfrac{1}{2}\{\hat{J}_x, \hat{J}_z\},$$
$$\hat{Q}_2^{(-)} = 2\sqrt{3}c_0\tfrac{1}{2}\{\hat{J}_x, i\hat{J}_y\}, \tag{3.9}$$

with

$$c_0 = \sqrt{\frac{5}{16\pi}}\frac{q_0}{j(j+1)}, \tag{3.10}$$

where q_0 is a constant with dimension $[L^2]$. We now assume that $\hat{J}_x$ in eq. (3.9) can be replaced by an aligned angular momentum i_x when we deal with the lowest-energy quasiparticle states. Making use of the relation (2.9) we then obtain

$$\hat{Q}_2^{(-)(\mathrm{qp})} \simeq -(-1)^{I_i-j}\frac{|\Delta E|}{\hbar\omega_{\mathrm{rot}}}\hat{Q}_1^{(-)(\mathrm{qp})}. \tag{3.11}$$

The relation (3.11) implies that the matrix elements $\langle \mathrm{f}|\hat{Q}_2^{(-)}|\mathrm{u}\rangle$ and $\langle \mathrm{f}|\hat{Q}_1^{(-)}|\mathrm{u}\rangle$ have the same sign. Numerical values of these matrix elements for the $N=90$ isotones are shown in fig. 1. These matrix elements are presented as ratios to $\langle \mathrm{f}|\hat{J}_z|\mathrm{u}\rangle$ for later

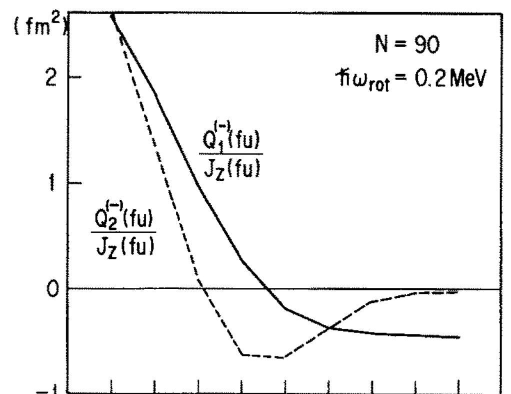

Fig. 1. Ratios of the single-quasiparticle matrix elements of quadrupole operators and the angular momentum between the lowest-energy signature-partner states in the $\pi h_{11/2}$ shell at $\hbar\omega_{\mathrm{rot}}=0.2$ MeV are plotted in (fm^2) as a function of the proton number for the $N=90$ isotones. The solid and the broken lines represent the ratios $\langle \mathrm{f}|\hat{Q}_1^{(-)}|u\rangle/\langle \mathrm{f}|\hat{J}_z|\mathrm{u}\rangle$ and $\langle \mathrm{f}|\hat{Q}_2^{(-)}|u\rangle/\langle \mathrm{f}|\hat{J}_z|\mathrm{u}\rangle$, respectively. The favoured signature is $r=+i$ in this case. Parameters used in these numerical calculations are $\beta^{(\mathrm{pot})}=0.20$, $\gamma^{(\mathrm{pot})}=0$ and $\Delta_{\mathrm{n}}=\Delta_{\mathrm{p}}=1.0$ MeV. Chemical potentials are determined such that the particle numbers are correctly reproduced at $\hbar\omega_{\mathrm{rot}}=0$.

convenience. Although there are some deviations in absolute values, the approximate phase relation (3.11) is fulfilled except for the case of ^{151}Pm where its value is close to zero.

Since we obtained a relation between $\langle f|\hat{Q}_2^{(-)}|u\rangle$ and $\langle f|\hat{Q}_1^{(-)}|u\rangle$, the phase relation for the signature dependence of $B(E2, \Delta I = 1)$ can be related to the signature splitting of the quasiparticle energies. From the commutators between h' and $(\hat{J}_z, i\hat{J}_y)$ in the prolate limit, we can derive the following identities between the quasiparticle matrix elements

$$-\Delta E\langle f|\hat{J}_z|u\rangle = \hbar\omega_{rot}\langle f|i\hat{J}_y|u\rangle\,,$$

$$-\Delta E\langle f|i\hat{J}_y|u\rangle = \hbar\omega_{rot}\langle f|\hat{J}_z|u\rangle + \sqrt{3}\alpha_0\langle f|\hat{Q}_1^{(-)}|u\rangle\,. \tag{3.12}$$

Using these identities we obtain the relation

$$\frac{\langle f|\hat{Q}_1^{(-)}|u\rangle}{\langle f|\hat{J}_z|u\rangle} = \frac{\hbar\omega_{rot}}{\sqrt{3}\alpha_0}\left\{\left(\frac{\Delta E}{\hbar\omega_{rot}}\right)^2 - 1\right\}. \tag{3.13}$$

We, therefore, conclude that the gamma-vibrational contributions make $B(E2, f \to u)$ larger than $B(E2, u \to f)$ when $(\Delta E/\hbar\omega_{rot})^2$ is smaller than unity and Q_0 is positive. Note that when we cannot neglect $\Delta E/\hbar\omega_{\gamma(-)}$, the cross terms, which are usually small, between the $K = 1$ and the $K = 2$ contributions remain. Since the transition amplitudes $T_1^{(-)}$ and $T_2^{(-)}$ associated with the gamma-vibrational phonon always have the same sign numerically, the signature dependence originated from these cross terms is also determined by eq. (3.13).

4. Nucleon-number dependence of the signature-dependent effects

The origin of the signature-dependent effects of the gamma vibration on the E2-transition matrix elements has been clarified in the preceding section. We discuss in this section the nucleon-number dependence of this effect and present the result of numerical calculations.

The signature splitting ΔE appearing in eq. (3.13) depends on the nucleon number and the rotational frequency in a rather complicated way, it depends also on the equilibrium shapes (β, γ) and the pairing gaps (Δ_p, Δ_n). Of course properties of the gamma-vibrational modes also depend on the nucleon-number and the rotational frequency. In order to see the shell-filling dependence of the signature-dependent effects avoiding the complications associated with the changes in (β, γ) and (Δ_p, Δ_n), we have performed numerical calculations for the $N = 90$ isotones with constant deformation parameters $\beta^{(pot)} = 0.20$, $\gamma^{(pot)} = 0$ and $\Delta_p = \Delta_n = 1.0$ MeV. The quadrupole-force strengths are fixed such that we obtain $\hbar\omega_{\gamma(\pm)} = 0.8$ MeV, $\hbar\omega_\beta = 1.0$ MeV and $\hbar\omega_{NG(\pm)} = 0$ at $\hbar\omega_{rot} = 0$ for each nucleus. The pairing-force strengths and the chemical potentials are fixed such that the above-mentioned pairing gaps and the correct nucleon numbers are reproduced at $\hbar\omega_{rot} = 0$ for each nucleus.

The signature splitting ΔE is smaller than the rotational frequency $\hbar\omega_{rot}$ when the chemical potential lies higher than the mid $\pi h_{11/2}$ shell region to which experimental data for $B(E2, \Delta I = 1)$ are available. On the other hand, when the chemical potential lies low in the $\pi h_{11/2}$ shell region, the ratio $\Delta E/\hbar\omega_{rot}$ can become larger than unity*. This is because the decoupling matrix elements $\langle f|\hat{J}_x|f\rangle$ and $\langle u|\hat{J}_x|u\rangle$ which already exist at $\hbar\omega_{rot} = 0$ greatly contribute to ΔE in such cases. Numerical examples can be seen in figs. 19 and 20 of ref. [5]). Numerical results for the ratio $\Delta E/\hbar\omega_{rot}$ for the $N = 90$ isotones are shown in fig. 2. The ratio decreases smoothly as the $\pi h_{11/2}$ shell is filled and crosses unity around the ^{151}Pm nucleus.

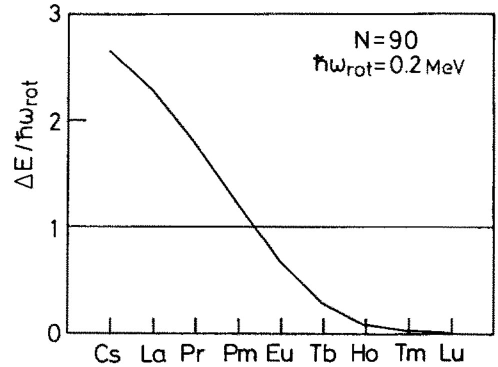

Fig. 2. Ratios of the signature splitting of the quasiparticle energies $\Delta E = E_u - E_f$ and the rotational frequency $\hbar\omega_{rot} = 0.2$ MeV. Parameters used are the same as in fig. 1.

The ratios $B(E2, f\to u)/B(E2, u\to f)$ obtained by using the exactly diagonalized wave functions, in which the gamma vibration with $r = +1$ is also taken into account, are presented in fig. 3. As is expected from eq. (3.13) and fig. 2, the sign of the signature-dependent effect is inverted in the vicinity of the ^{149}Pr nucleus. Here we note that small contributions from the odd-quasiparticle part (in the second line in eq. (3.5)) and from small but finite expectation values of Q_2 (in the first line in eq. (3.5)) due to the breaking of the self-consistency between the shape of the potential (given by $\beta^{(pot)}$ and $\gamma^{(pot)}$) and that of the density are contained in this result; typically the former is a few percent of each $B(E2)$ value and Q_2/Q_0 is 0.001 in the latter.

The result shown in fig. 3 is consistent with that of all the previous works: (i) When the Fermi surface lies at the mid $\pi h_{11/2}$ shell region, the signature splitting is smaller than the magnitude of the rotational frequency. In this case, the gamma-vibrational contributions enhance $B(E2, f\to u)$. This is the case discussed by Ikeda [2]) and in our previous works [5,6]). (ii) When the Fermi surface lies low in the $\pi h_{11/2}$ shell, the signature splitting is larger than the rotational frequency. In this case, the

* The statement made in ref. [6]) that the ratio $\Delta E/\hbar\omega_{rot}$ is less than unity applies only for the $Z \geqslant 63$ region considered in this reference.

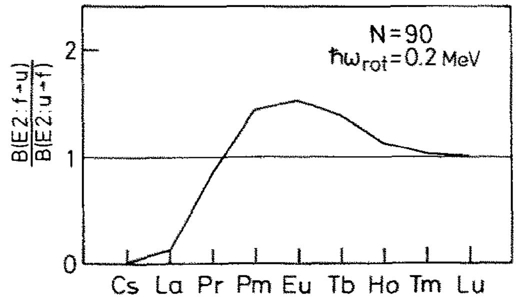

Fig. 3. Ratios of $B(\mathrm{E2}, \Delta I = 1)$. The doubly stretched quadrupole-force strengths are determined such that the vibrational frequencies $\hbar\omega_{\gamma(\pm)}$, $\hbar\omega_{\beta}$ and $\hbar\omega_{\mathrm{NG}(\pm)}$ become 0.8 MeV, 1.0 MeV and 0, respectively, at $\hbar\omega_{\mathrm{rot}} = 0$ in 3-major shell calculation. The pairing-force strengths are determined so as to reproduce $\Delta_{\mathrm{p}} = \Delta_{\mathrm{n}} = 1.0$ MeV. Other parameters are the same as fig. 1. In the cases of ^{157}Ho and ^{159}Tm, the signature dependence is weaker than that in figs. 9 and 10 of ref. [5]). This is because a smaller energy gap is used in the present calculation in order to approximately reproduce the average value of the experimental odd–even mass differences in the ^{145}Cs-^{161}Lu region.

gamma-vibrational contributions enhance $B(\mathrm{E2}, \mathrm{u} \to \mathrm{f})$. This is the case discussed by Onishi *et al.* [4]).

The phase relation (3.13) can be understood also from another point of view: The pairing factor appearing in the $a^{+}a$ part of the operators $\hat{J}_z$ and $i\hat{J}_y$ are $(u\bar{u} + v\bar{v})$, while the corresponding part of the quadrupole operators $\hat{Q}_K^{(-)}$'s contain the factor $(u\bar{u} - v\bar{v})$. The relative sign between these factors changes as the occupation number in the $\pi h_{11/2}$ shell changes.

5. Concluding remarks

We have studied the signature-dependent effects of the gamma-vibrational contributions on $B(\mathrm{E2}, \Delta I = 1)$ in rotating odd-A nuclei from the microscopic point of view. The signature dependence is brought about directly by the $K = 2$ components of the quadrupole operators both in the rotational part and the vibrational part in the E2-transition matrix elements. We have established the phase rule for the signature-dependent effects associated with the gamma vibration in relation to the signature splitting of the quasiparticle energies. The magnitude of the signature splitting changes depending on the occupation probabilities in the $\pi h_{11/2}$ shell. Thus the signature-dependent effects are expected to exhibit a characteristic nucleon-number dependence: Namely, $B(\mathrm{E2}, \mathrm{f} \to \mathrm{u})$ is enhanced in the mid $\pi h_{11/2}$ shell region while $B(\mathrm{E2}, \mathrm{u} \to \mathrm{f})$ is enhanced in the beginning of the $\pi h_{11/2}$ shell region, if the effects of the static triaxiality are small. Needless to say, the phase rule found in this paper applies also to any unique-parity band other than the $\pi h_{11/2}$ orbital.

Experimental data for the E2-transition rates in nuclei whose Fermi surfaces lie low in the $\pi h_{11/2}$ shell have not been available yet. These data are desired to test our theoretical prediction experimentally.

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