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SPATIAL STRUCTURE OF QUARK COOPER PAIRS

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The spatial structure of Cooper pairs with quantum number color $\mathbf{3}^*$, $I = J = L = S = 0$ in *ud* 2 flavor quark matter is studied by solving the gap equation in full momentum range without the weak coupling approximation. Although the gap at the Fermi surface and the coherence length depend on density weakly, the shape of the pair wave function varies strongly with density. This result indicates that quark Cooper pairs become more bosonic at higher densities.

Color superconducting phases of strongly interacting matter at high density and low temperature are attracting much attention recently. They were studied first as an example of pair condensation in relativistic many-body systems by Bailin and Love in early 80's.[1] Iwasaki and Iwado's work on a 1 flavor system[2] was the first study of the realistic SU(3) color system. Since the works of Rapp *et al.*[3] and Alford *et al.*,[4] color superconductivity has been studied extensively. The $^1S_0$ ($J = L = S = 0$) state in *ud* 2 flavor case and the color-flavor-locked state in *uds* 3 flavor case, respectively, have been understood to be the most favored channels.

The purpose of this talk is to visualize the spatial structure, in particular its density dependence, of quark Cooper pairs, which has not been discussed, to the author's knowledge. To this end, calculations are performed for Cooper pairs of the simplest form at zero temperature under an instantaneous approximation while full **k**-dependence is retained. That is, we allow strong coupling in the sense that momenta far from the Fermi surface also contribute. The formulation is based on Refs.[2,5]

We start from deriving the equation for $\Delta(k)$ representing the gap for color $\mathbf{3}^*$, isosinglet $^1S_0$ pairs composed of two quarks which are time-reversal conjugate to each other with respect to space and spin,

$$\Delta_{\mathbf{k}sfi,\mathbf{k}'s'f'i'} = (-1)^{\frac{1}{2}-s}\delta_{\mathbf{k},-\mathbf{k}'}\delta_{s,-s'}\epsilon_{ff'}\hat{\epsilon}_{ii'}\Delta(k)\,,$$
$$\epsilon_{ff'} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},\ \hat{\epsilon}_{ii'} = \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}, \qquad (1)$$

where s, f and i denote spin, flavor and color, respectively. The gap equation

is obtained by diagonalizing the 12×12 Hamiltonian matrix,

$$\begin{pmatrix} E_k - \mu & \Delta \\ -\Delta^* & -(E_k - \mu) \end{pmatrix},$$
$$E_k = \sqrt{\mathbf{k}^2 + M_q^2},\ \mu = E_{k_\mathrm{F}}\,, \qquad (2)$$

and expressing the pair condensate in terms of the resulting Bogoliubov coefficients, as

$$\Delta(p) = -\frac{1}{8\pi^2}\int_0^\infty \bar{v}(p,k)\frac{\Delta(k)}{E'(k)}k^2dk\,,$$
$$E'(k) = \sqrt{(E_k - E_{k_\mathrm{F}})^2 + 3\Delta^2(k)}\,. \qquad (3)$$

As for the residual interaction, we adopt the one gluon exchange interaction with a Debye screening mass in the electric part, $m_\mathrm{E}^2 = \frac{4}{\pi}\alpha_s\mu^2$,

$$\bar{v}(p,k) = -\frac{\pi}{3}\alpha_s\frac{1}{pkE_pE_k}$$
$$\times\left(\left(2E_pE_k + 2M_q^2 + p^2 + k^2 + m_\mathrm{E}^2\right)\ln\left(\frac{(p+k)^2 + m_\mathrm{E}^2}{(p-k)^2 + m_\mathrm{E}^2}\right)\right.$$
$$\left. +2\left(6E_pE_k - 6M_q^2 - p^2 - k^2\right)\ln\left|\frac{p+k}{p-k}\right|\right). \qquad (4)$$

The running coupling constant is taken from Ref.[6] as in Refs.,[2,5]

$$\alpha_s(\mathbf{q}^2) = \frac{4\pi}{9}\frac{1}{\ln\left(\frac{q_\mathrm{max}^2 + q_\mathrm{c}^2}{\Lambda_\mathrm{QCD}^2}\right)}\,,$$
$$\mathbf{q} = \mathbf{p} - \mathbf{k}\,,\ q_\mathrm{max}^2 = \max\{\mathbf{p}^2, \mathbf{k}^2\}\,, \qquad (5)$$

with parameters $q_\mathrm{c}^2 = 1.5\Lambda_\mathrm{QCD}^2$, $\Lambda_\mathrm{QCD} = 0.4$ GeV. As for the quark mass, $M_q = 10$ MeV is adopted according to Ref.[2]

First we discuss the gap at the Fermi surface and the coherence length ξ as functions of the Fermi momentum k_F, which is related to the baryon density as $\rho = 2k_\mathrm{F}^3/3\pi^2$ in the present 2 flavor case. The coherence length is defined as,[7]

$$\xi = \left(\frac{\int_0^\infty |\frac{d\phi}{dk}|^2k^2dk}{\int_0^\infty |\phi|^2k^2dk}\right)^{1/2}, \qquad (6)$$

in the strong coupling case, in terms of the pair wave function,

$$\phi(k) = \frac{1}{2}\frac{\Delta(k)}{E'(k)}\,. \qquad (7)$$

In Fig.1(a), $\Delta(k_F)$ is graphed for $k_F = 1.5 - 3.25$ fm^{-1}, which corresponds to $\rho/\rho_0 \simeq 1.5 - 15$ if the normal density of symmetric ($N = Z$) nuclear matter is defined as $\rho_0 = 2(k_F)_0^3/3\pi^2$, $(k_F)_0 = 1.30$ fm^{-1}.[8] This shows very weak k_F-dependence; the superconducting phase survives up to practically infinite density. The coherence length is shown in Fig.1(b). This can be compared with the Pippard length in the weak coupling theory, $\xi_0 = k_F/\pi\Delta(k_F)\mu \simeq 1/\pi\Delta(k_F)$; the *magnitudes* of ξ are approximated fairly well by ξ_0 whereas the k_F*-dependence* is different. The average interparticle distance $d = \left(\frac{\pi^2}{2}\right)^{1/3}/k_F$, derived from $\rho_q = 3\rho = 1/d^3$, is also shown in the figure. The magnitudes of ξ and d are very similar to each other as in the nucleon-nucleon case,[9] whereas ξ is 3 – 4 orders of magnitude larger than d in metals. This indicates strong coupling feature in the sense that quark Cooper pairs are compact and therefore bosonic. That is, fermion exchange is less when their mutual overlap is small.

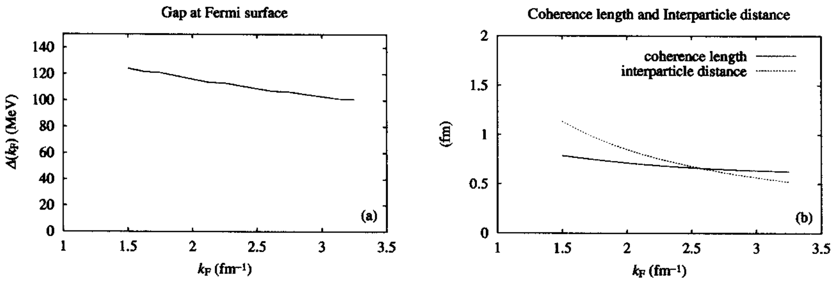

Figure 1: (a) Pairing gap at the Fermi surface, and (b) coherence length and average interquark distance, as functions of the Fermi momentum k_F.

Next, we turn to k-dependence at each k_F. The compactness of Cooper pairs mentioned above indicates spreading of $\phi(k)$ in k space. Figure 2(a) shows that the width of $\phi(k)$, which corresponds to $1/\xi$, is almost k_F-independent as mentioned above since the quasiparticle energy $E'(k)$ in the denominator in Eq.(7) grows as k goes away from k_F.

Finally, we Fourier-transform $\phi(k)$ to

$$\phi(r) = \frac{1}{2\pi^2}\int_0^\infty \phi(k) j_0(kr) k^2 dk\,, \tag{8}$$

in order to look into the spatial structure of quark Cooper pairs more closely. Although the coherence length ξ, corresponding to the root mean square distance with respect to this wave function, is almost k_F-independent, the shape

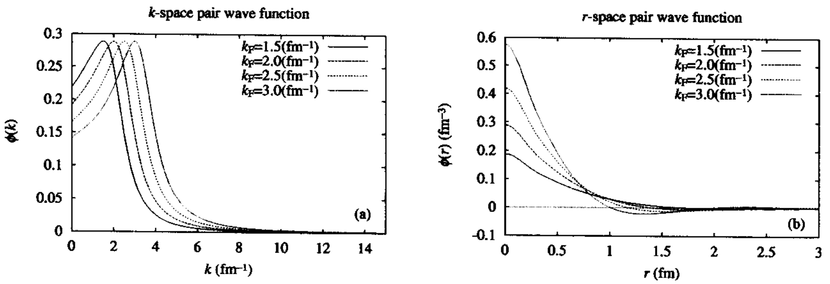

Figure 2: Pair wave function as functions of (a) the momentum k and (b) the distance r calculated at $k_F = 1.5 - 3.0$ fm^{-1}.

of $\phi(r)$ is strongly k_F-dependent; large-k components in high-density cases bring nodes in r-space. In other words, $\phi(r)$ is diffuse at low densities. This indicates that quark Cooper pairs become more bosonic at higher densities.

To summarize, we have studied numerically the spatial structure of quark Cooper pairs in the color $\mathbf{3}^*$, isosinglet $^1S_0$ channel by solving the gap equation in full momentum range. The resulting coherence length is almost density independent as well as the gap at the Fermi surface and is of magnitudes similar to the average interquark distance. The dependence of the pair wave function (condensate) on the relative momentum and distance between two quarks that form a Cooper pair has been presented. This indicates that quark Cooper pairs become more bosonic at higher densities although the coherence length is almost density independent.

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