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Spatial Structure of Quark Cooper Pairs in a Color Superconductor

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Abstract

Spatial structure of Cooper pairs with quantum numbers color $\mathbf{3}^*$, $I = J = L = S = 0$ in ud 2 flavor quark matter is studied by solving the gap equation and calculating the coherence length in full momentum range without the weak coupling approximation. Although the gap at the Fermi surface and the coherence length depend on density weakly, the shape of the r-space pair wave function varies strongly with density. This result indicates that quark Cooper pairs become more bosonic at higher densities.

1. Introduction

Color superconducting phases of strongly interacting matter at high density and low temperature [1, 2, 3] are attracting much attention recently. They were studied first as an example of pair condensation in relativistic many-body systems by Bailin and Love in the early 80's [4] (see also Ref.[5]). They also mentioned the nucleon-nucleon pairing as another example; its detailed study was begun in the early 90's [6] and is developing recently [7, 8, 9, 10]. A study of the quark-quark pairing in an SU(2) color model was done at almost the same time [11] (see also Refs.[12, 13]). Iwasaki and Iwado's work on a 1 flavor system [14] was the first study of the realistic SU(3) color system (see also Ref.[15]). Since the works of Rapp et al. [16] and Alford et al. [17] on ud 2 flavor system, color superconductivity has been studied extensively. The $^1S_0$ ($J = L = S = 0$) state in the 2 flavor case [18] and the color-flavor-locked state in uds 3 flavor case [19, 20, 21, 22, 23], respectively, have been understood to be the most favored channels. The magnetic interaction has been shown to be responsible for these pair condensations [24, 25, 26, 27, 28, 29]. Their astrophysical consequences were also studied [30, 31].

The purpose of this talk is to visualize the spatial structure, in particular its density dependence, of quark Cooper pairs, which has not been discussed, to the author's knowledge. This is done for Cooper pairs of the simplest form;

color **3***, isosinglet $^1S_0$ pairs in the 2 flavor system. To this end, calculations are performed for zero temperature under an instantaneous approximation while full **k**-dependence is retained. That is, we allow strong coupling in the sense that momenta far from the Fermi surface also contribute.

2. Formulation

We start from deriving the gap equation for $\Delta(k)$ representing the gap for color **3***, isosinglet $^1S_0$ pairs composed of two quarks which are time-reversal conjugate to each other with respect to space and spin,

$$\Delta_{\mathbf{k}sfi,\mathbf{k}'s'f'i'} = (-1)^{\frac{1}{2}-s}\delta_{\mathbf{k},-\mathbf{k}'}\delta_{s,-s'}\epsilon_{ff'}\hat{\epsilon}_{ii'}\Delta(k)\,,$$
$$\epsilon_{ff'} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \hat{\epsilon}_{ii'} = \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}, \tag{1}$$

where s, f and i denote spin, flavor and color, respectively. Note that time-reversal is represented by T times complex conjugate, and $T = -iC\gamma^5$. We work in a Hamiltonian formalism since it is more convenient than the Gor'kov formalism [32] in the present study in which only the pair condensate is considered. The latter might be more suitable when the coupling between the pair condensate and the chiral condensate [17] is considered. Such a formalism has already been developed for the nuclear system [7, 9] although the N-$\bar{N}$ condensate is negligibly small [9] due to a large nucleon mass. The resulting 12×12 Hamiltonian matrix,

$$\begin{pmatrix} E_k - \mu & \Delta \\ -\Delta^* & -(E_k - \mu) \end{pmatrix},$$
$$E_k = \sqrt{\mathbf{k}^2 + M_q^2}, \; \mu = E_{k_\mathrm{F}}\,, \tag{2}$$

is easily block-diagonalized to two 6×6 matrices by inspection. Then they are fully diagonalized along the lines of the 1 flavor case studied in Ref.[14]. By expressing the pair condensate in terms of the Bogoliubov coefficients, we obtain

$$\Delta(p) = -\frac{1}{8\pi^2}\int_0^\infty \bar{v}(p,k)\frac{\Delta(k)}{E'(k)}k^2 dk\,,$$
$$E'(k) = \sqrt{(E_k - E_{k_\mathrm{F}})^2 + 3\Delta^2(k)}\,, \tag{3}$$

here the factor 3 reflects the fact that quarks have three colors.

As for the residual interaction, we adopt the one gluon exchange interaction with the leading order screening. Although the gluon propagator therein contains a gauge-dependent term in general, it vanishes due to the equation of motion of the external quark. Then we proceed to a static (instantaneous) approximation [14, 15, 18]. At this step, the dynamic magnetic screening drops. By performing a spin average and an angle integration to project out the S-wave component, we obtain

$$\bar{v}(p,k) = -\frac{\pi}{3}\alpha_s\frac{1}{pkE_pE_k}$$
$$\times\left(\left(2E_pE_k + 2M_q^2 + p^2 + k^2 + m_\mathrm{E}^2\right)\ln\left(\frac{(p+k)^2 + m_\mathrm{E}^2}{(p-k)^2 + m_\mathrm{E}^2}\right)\right.$$
$$\left. +2\left(6E_pE_k - 6M_q^2 - p^2 - k^2\right)\ln\left|\frac{p+k}{p-k}\right|\right),$$
$$m_\mathrm{E}^2 = \frac{4}{\pi}\alpha_s\mu^2\,. \tag{4}$$

The running coupling constant is taken from Ref.[33] as in Refs.[15, 18],

$$\alpha_s(\mathbf{q}^2) = \frac{4\pi}{9}\frac{1}{\ln\left(\frac{q_\mathrm{max}^2 + q_c^2}{\Lambda_\mathrm{QCD}^2}\right)}\,,$$
$$\mathbf{q} = \mathbf{p} - \mathbf{k}\,, \; q_\mathrm{max} = \max\{p, k\}\,, \tag{5}$$

with parameters $q_c^2 = 1.5\Lambda_\mathrm{QCD}^2$, $\Lambda_\mathrm{QCD} = 0.4$ GeV.

3. Density Dependence

In the following, we present numerical results in three steps. These can be regarded as an upper limit of the pair correlation since the magnetic interaction is also screened if the retardation is allowed. As for the quark mass, $M_q = 10$ MeV is adopted according to Ref.[14]. First we discuss the gap at the Fermi surface $\Delta(k_\mathrm{F})$ and the coherence length ξ as functions of the Fermi momentum k_F, which is related to the baryon density as $\rho = 2k_\mathrm{F}^3/3\pi^2$ in the present 2 flavor case. The coherence length is defined as [34],

$$\xi = \left(\frac{\int_0^\infty |\frac{d\phi}{dk}|^2 k^2 dk}{\int_0^\infty |\phi|^2 k^2 dk}\right)^{1/2}, \tag{6}$$

in the strong coupling case, in terms of the pair wave function,

$$\phi(k) = \frac{1}{2}\frac{\Delta(k)}{E'(k)}\,, \tag{7}$$

which is identical to the pair condensate up to phase factors. In Fig.1.(a), $\Delta(k_\mathrm{F})$ is graphed for $k_\mathrm{F} = 1.5 - 3.25$ fm^{-1}, which corresponds to $\rho/\rho_0 \simeq 1.5 - 15$ if the normal density of symmetric ($N = Z$) nuclear matter is defined as $\rho_0 =$

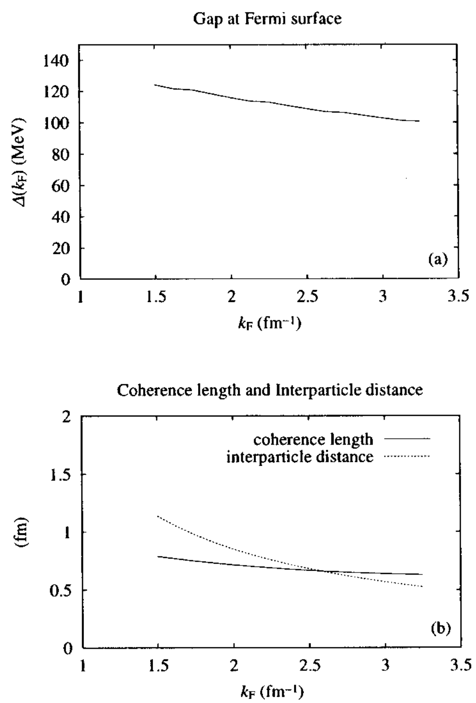

Fig. 1. (a) Pairing gap at the Fermi surface, and (b) coherence length and average interquark distance, as functions of the Fermi momentum k_F.

$2(k_F)_0^3/3\pi^2$, $(k_F)_0 = 1.30$ fm^{-1} [35]. This shows very weak k_F-dependence; the superconducting phase survives up to practically infinite density; for example, $\Delta(k_F = 60\,\text{fm}^{-1}) \simeq 50$ MeV. We confirmed that the gap is predominantly brought about by the magnetic interaction; about 50% of the total gap remains when the electric interaction is cut artificially while only 5 – 10% remains when the magnetic part is cut. This magnetic dominance can also be deduced from the strong k_F-dependence of $\bar{v}(k_F, k)$ (Fig.2.(c) below) except at $k \simeq k_F$ where the magnetic interaction gives very strong attraction irrespective of k_F, in contrast to the weak k_F-dependence of $\Delta(k_F)$. The coherence length is shown in Fig.1.(b). This can be compared with the Pippard length in the weak coupling theory, $\xi_0 = k_F/\pi\Delta(k_F)\mu \simeq 1/\pi\Delta(k_F)$ (see for example, Ref.[36]); the magnitudes of ξ are approximated fairly well by ξ_0 whereas the k_F-dependence is different. The average interparticle distance $d = \left(\frac{\pi^2}{2}\right)^{1/3}/k_F$, derived from $\rho_q = 3\rho = 1/d^3$, is also shown in the figure. The magnitudes of ξ and d are very similar to each other as in the nucleon-nucleon case [10], whereas ξ is 3 – 4 orders of magnitude larger than d in metals. This indicates strong coupling feature in the sense that quark Cooper pairs are compact and therefore bosonic. That is, fermion exchange is less when their mutual overlap is small. If M_q is changed to 100 MeV and 300 MeV, for example, $\Delta(k_F)$ decreases to 90 – 98 % and 54 – 81 % of the original values, respectively, for $k_F = 1.5 - 3.25$ fm^{-1}, whereas the changes in ξ are negligibly small.

Another quantity which can be compared with ξ is the London penetration depth λ_L, which is defined by $\sqrt{\mu/4\pi e_s^2 \rho_s}$ in the relativistic case, where e_s and ρ_s are the electric charge of the Cooper pair and the density of the superconducting component, respectively. This expression can be derived from the relativistic Ginzburg-Landau theory developed in Ref.[4] (see also Refs.[30, 25]). In the present ud 2 flavor case, $\lambda_L \simeq 20 - 8$ fm for $k_F = 1.5 - 3.25$ fm^{-1} is obtained by adopting $e_s = \frac{2}{3}e + (-\frac{1}{3})e$ and $\rho_s = \frac{2}{3}\rho_q$. Although Bailin and Love concluded that quark matter is a superconductor of the first kind, $\xi > \sqrt{2}\lambda_L$, based on their estimate, $\Delta(k_F) \sim 1$ MeV [4], the present quantitative study shows that quark matter is a superconductor of the second kind at these densities as pointed out in Ref.[30]. At asymptotically high densities, however, it changes to a superconductor of the first kind since the penetration depth decreases rapidly as density increases. One should note that these discussion applies only to a part of magnetic field since the original photon and a gluon combine to a massless 'rotated' photon although both the electromagnetic U(1) and the color SU(3) break in color superconductors, as recently discussed in Ref.[37].

4. Spatial Structure

Next, we turn to k-dependence at each k_F. The compactness of the Cooper pairs mentioned above indicates spreading of $\Delta(k)$ (Fig.2.(a)) and $\phi(k)$ (Fig.2.(b)) in k-space. Figure 2.(a) shows that $\Delta(k)$ spreads to larger k at higher densities as expected, while the width of $\phi(k)$ in Fig.2.(b), which corresponds to $1/\xi$, is almost k_F-independent as mentioned above since the quasiparticle energy $E'(k)$ in the denominator in Eq.(7) grows as k goes away from k_F. The asymmetric shapes of $\phi(k)$ reflect the smallness of ξ. Its width is similar to that of $\bar{v}(k_F, k)$ in Fig.2.(c).

Finally, we Fourier-transform $\phi(k)$ to

$$\phi(r) = \frac{1}{2\pi^2}\int_0^\infty \phi(k) j_0(kr) k^2 dk\,, \tag{8}$$

where $j_0(kr)$ is a spherical Bessel function, in order to look into the spatial structure of quark Cooper pairs more closely. This quantity has already been discussed

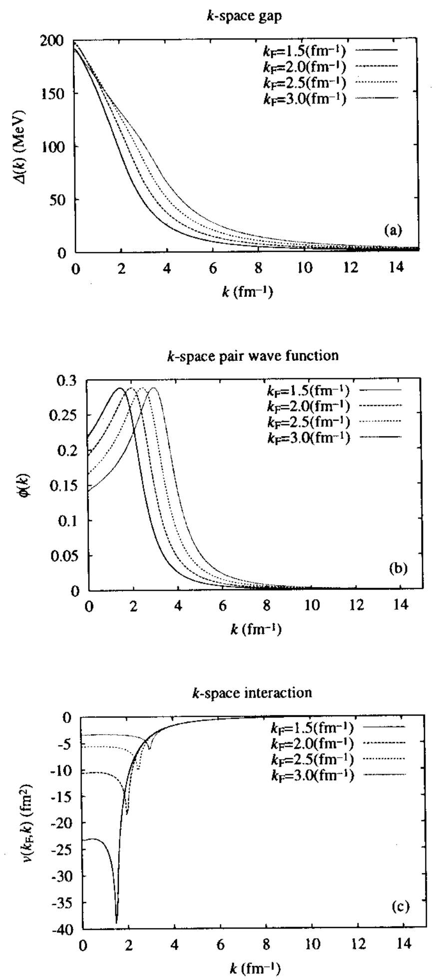

Fig. 2. (a) Gap function, (b) pair wave function, and (c) matrix element $\bar{v}(k_F, k)$, as functions of the momentum k, calculated at $k_F = 1.5 - 3.0\ \mathrm{fm}^{-1}$. Note that, in (c), $k = k_F$ is excluded since at this point the diverging magnetic interaction does not contribute to the gap equation [38].

for nucleon Cooper pairs in non-relativistic [39, 40] and relativistic [10] studies. Although the coherence length ξ, corresponding to the root mean square distance with respect to this wave function, is almost k_F-independent, the shape of $\phi(r)$ is strongly k_F-dependent; large-k components in high-density cases bring nodes in r-space. Conversely, $\phi(r)$ is diffuse at low densities. In other words, quark Cooper pairs become more bosonic at higher densities.

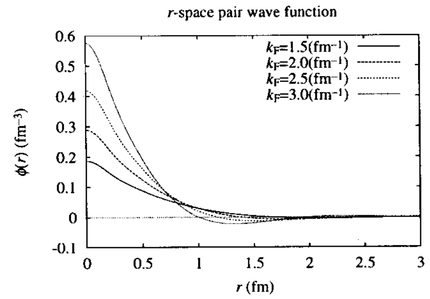

Fig. 3. Pair wave function as a function of the distance r, calculated at $k_F = 1.5 - 3.0\ \mathrm{fm}^{-1}$.

5. Summary

To summarize, we have studied numerically the spatial structure of quark Cooper pairs in the color $\mathbf{3}^*$, isosinglet $^1S_0$ channel by solving the gap equation in full momentum range. Although the long-range magnetic interaction is predominantly responsible for the quark-quark pairing, the gap function spreads also in momentum space. The resulting coherence length is almost density independent as well as the gap at the Fermi surface and is of magnitudes similar to the average interquark distance. The dependence of the pair wave function (condensate) on the relative momentum and on the distance between two quarks that form a Cooper pair has been presented. This indicates that quark Cooper pairs become more bosonic at higher densities although the coherence length is almost density independent.

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List of Symbols

k=Magnitude of Quark 3-momentum
k_F=Quark Fermi Momentum
M_q=Current Quark Mass
E_k=Quark Single-particle Energy
μ=Quark Chemical Potential
$\Delta(k)$=u-d Pairing Gap
$E'(k)$=Quark Quasiparticle Energy
$\mathbf{q}$=Momentum Transfer in One Gluon Exchange Interaction
$\alpha_s(\mathbf{q}^2)$=QCD Running Coupling Constant
q_c=Cutoff Momentum to Avoid Infrared Divergence
Λ_{QCD}=QCD Mass Scale
ξ=Coherence Length of Quark-Quark Pair Correlation
$\phi(k)$=Quark Pair Wave Function
$\phi(r)$=Its Fourier Transform

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