Relativistic tilted axis cranking

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We apply the Relativistic Mean Field (RMF) model to the Tilted Axis Cranking (TAC). As a test of our numerical code, we first investigate the ground state rotational band in ^{24}Mg.

Introduction

Recently obtained data of the so-called shears bands in the proton rich Pb isotopes are well described within the framework of the TAC model. In the TAC approximation, the nucleus rotates around the axis which is tilted from its principal axes. In the shears mechanism, the magnetic dipole vector, which arises from few proton particles(holes) and few neutron holes(particles) in high j orbitals, rotates around the total angular momentum vector. At the band head, the proton and the neutron angular momenta are almost perpendicular. With increasing the rotational frequency, these angular momenta align toward the total angular momentum. Consequently, the direction of the total angular momentum does not change so much and regular rotational bands are formed in spite of the fact that the density distribution of the nucleus is almost spherical. These kinds of rotation are called magnetic rotation in order to distinguish from the usual collective rotation in well-deformed nuclei (called electric rotation). Magnetic rotations are also observed in other regions such as $A \sim 140$, 110 and recently 80 regions.

From the theoretical side, such tilted axis rotation has been well examined by the pairing + QQ model, shell model and particle-rotor model. Qualitatively, these models can explain the observed trends. For quantitative description, however, fully microscopic self-consistent calculations are desirable. To this time, only few investigations by using such models have been done. Therefore, in this work, we develop the RMF code which can be applied to the study of the tilted axis rotation. The RMF model has been successfully applied to many phenomena in nuclear physics in the low energy region. Rotating nuclei are also examined within the context of the RMF model. However, such applications are limited only to the 1-dimensional case (PAC: Principal axis cranking) so far. Because the RMF model is a phenomenological one, its applicability should be examined further. This is another motivation why we try to apply the RMF model to such tilted axis rotation.

Formulation

The starting point of the RMF model is the following Lagrangian, which contains the nucleon and several kinds of meson fields, such as σ-, ω- and ρ-mesons, together with the photon fields (denoted by A) mediating the Coulomb interaction:

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\rho + \mathcal{L}_A + \mathcal{L}_{\rm int} + \mathcal{L}_{{\rm NL}\sigma}.$$

$\mathcal{L}_{\rm int}$ is the interaction part between nucleons and mesons:

$$\begin{aligned}\mathcal{L}_{\rm int} &= g_\sigma\overline{\psi}\psi\sigma - g_\omega\overline{\psi}\gamma^\alpha\psi\omega_\alpha \\ &\quad - g_\rho\overline{\psi}\gamma^\alpha\overrightarrow{\tau}\psi\cdot\overrightarrow{\rho}_\alpha - e\overline{\psi}\gamma^\alpha\frac{1-\tau_3}{2}\psi A_\alpha.\end{aligned}$$

In the standard applications, non-linear self interactions among the σ-mesons,

$$\mathcal{L}_{{\rm NL}\sigma} = \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4,$$

are also included.

For application to rotating nuclei within the cranking assumption, it is necessary to write the Lagrangian in a uniformly rotating frame which rotates around some fixed axis with a constant angular velocity $\mathbf{\Omega} = (\Omega_x, \Omega_y, \Omega_z)$, from which the equations of motion in this frame can be obtained. Because the rotating frame is not an inertial frame, a fully covariant formulation is desirable, and we obtain this using the technique of general relativity known as tetrad formalism. The procedure is as follows. First, according to tetrad formalism, we can write the Lagrangian in the non-inertial frame represented by the metric tensor $g_{\mu\nu}$. Then the variational principle gives the equations of motion in this non-inertial frame. Finally, substituting the metric tensor of the uniformly rotating frame leads to the desired equations of motion. In the 1-dimensional case, the detail was already shown in Ref. 1. Extension to the 3-dimensional case is straightforward. The only difference is that the metric tensor substituted into the general equations is now truly 3-dimensional one which can be constructed from the relation between the laboratory coordinate (T, X, Y, Z) and the rotating one (t, x, y, z),

$$\begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} T \\ (c_1c_2c_3 - s_1s_3)X + (s_1c_2c_3 + c_1s_3)Y - c_3s_2Z \\ (-c_1c_2s_3 - s_1c_3)X + (-s_1c_2s_3 + c_1c_3)Y - s_3s_2Z \\ c_1s_2X + s_1s_2Y - c_2Z \end{pmatrix},$$

where we abbreviate $\sin\theta_{1,2,3}$ and $\cos\theta_{1,2,3}$ as $s_{1,2,3}$ and $c_{1,2,3}$, respectively. The metric tensor is then

$$g_{\mu\nu}(x) = \begin{pmatrix} 1-(\boldsymbol{\Omega}\times\boldsymbol{x})^2 & -(\boldsymbol{\Omega}\times\boldsymbol{x}) \\ -(\boldsymbol{\Omega}\times\boldsymbol{x}) & -1 \end{pmatrix},$$

here the rotational frequency is connected with the Euler angles $(\theta_1, \theta_2, \theta_3)$ as

$$\boldsymbol{\Omega} = \begin{pmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{pmatrix} = \begin{pmatrix} -s_2c_3 & s_3 & 0 \\ s_2s_3 & c_3 & 0 \\ c_2 & 0 & 1 \end{pmatrix} \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{pmatrix}.$$

The resulting equations are

$$\begin{aligned}
&\Big\{\boldsymbol{\alpha}\cdot(\frac{1}{i}\boldsymbol{\nabla} - g_\omega\boldsymbol{\omega}(x)) + \beta(M - g_\sigma\sigma(x)) \\
&\qquad + g_\omega\omega^0(x) - \boldsymbol{\Omega}\cdot(\boldsymbol{L}+\boldsymbol{\Sigma})\Big\}\,\psi_i(x) = \epsilon_i\psi_i(x), \\
&\left\{-\boldsymbol{\nabla}^2 + m_\sigma^2 - (\boldsymbol{\Omega}\cdot\boldsymbol{L})^2\right\}\sigma(x) \\
&\qquad - g_2\sigma^2(x) + g_3\sigma^3(x) = g_\sigma\rho_s(x), \\
&\left\{-\boldsymbol{\nabla}^2 + m_\omega^2 - (\boldsymbol{\Omega}\cdot\boldsymbol{L})^2\right\}\omega^0(x) = g_\omega\rho_v(x), \\
&\left\{-\boldsymbol{\nabla}^2 + m_\omega^2 - (\boldsymbol{\Omega}\cdot(\boldsymbol{L}+\boldsymbol{S}))^2\right\}\boldsymbol{\omega}(x) = g_\omega\boldsymbol{j}_v(x),
\end{aligned}$$

where the ρ-meson and photon fields are omitted for simplicity, although they are included in the numerical calculation. These equations are the same as those obtained by the Munich group.[1,2]

Numerical code

The technique to solve the coupled equations of motion is essentially the same as that in the 1-dimensional case. The nucleon and meson fields are expanded in terms of the 3-dimensional harmonic oscillator eigenfunctions. In the TAC approximation, the signature quantum number is not conserved. Therefore, the parity is the only symmetry which is assumed in our code. One of the main differences from the 1-dimensional case is the Coriolis term for the nucleon fields which becomes $-\Omega_x J_x - \Omega_y J_y - \Omega_z J_z$. For simplicity, the Coriolis terms for the meson and photon fields, which appear in the Klein-Gordon equations, are neglected. It is known that these Coriolis terms for the meson and photon fields give very small contributions and do not affect the results, at least in the 1-dimensional case.

As a result of breaking the reflection symmetry with respect to the principal planes, the matrix elements of the Dirac Hamiltonian become complex numbers contrary to the 1-dimensional case. If we restrict ourselves to the 2-dimensional cranking in which the rotational axis is deviated from the principal axes but still in one of the principal planes, such components that make the matrix elements imaginary are small and can be neglected. In the 3-dimensional case, on the other hand, diagonalizing the complex Hamiltonian is necessary. Because we will also examine the tilted bands in triaxially deformed nuclei in future, which requires the 3-dimensional calculation, we decided to diagonalize the complex Hamiltonian in our code.

We also add the following three constraints,

$$-\lambda_x B_x - \lambda_y B_y - \lambda_z B_z,$$

here

$$B_x = -\sqrt{6}yz, B_y = -\sqrt{6}zx, B_z = -\sqrt{6}xy,$$

to the Dirac Hamiltonian, which are necessary to fulfill the condition of the principal frame,[3] that is, $\langle Q_{2\pm1}\rangle = 0$ and $\langle Q_{2-2}\rangle = \langle Q_{22}\rangle$. Actually we adopt the quadratic constraints;

$$\begin{aligned}
\lambda_x &= -C(\langle B_x\rangle - B_x^{(\text{required})}), \\
\lambda_y &= -C(\langle B_y\rangle - B_y^{(\text{required})}), \\
\lambda_z &= -C(\langle B_z\rangle - B_z^{(\text{required})}).
\end{aligned}$$

If we choose relatively large value of C and set $B_x^{(\text{required})} \sim B_y^{(\text{required})} \sim B_z^{(\text{required})} \sim 0$, the resulting values of $\langle B_x\rangle, \langle B_y\rangle$ and $\langle B_z\rangle$ become very small.

In the present code, the pairing correlations are not taken into account. They should be included for precise description of the properties of heavy and medium-heavy nuclei, although including the pairing correlations makes the calculation very time-consuming one. We will extend our code so as to include the pairing correlations within Relativistic Hartree Bogoliubov framework in near future.

Results for ^{24}Mg

As a check of the numerical code, we first investigate the ground state rotational band in ^{24}Mg. Although there is no evidence of appearance of such tilted axis rotation in the *sd*-shell nuclei, we think this nucleus is convenient for the purpose of checking our code. This is because for lighter nuclei, large model space is not required, that is, the cutoff parameters for expansions of the nucleon and meson fields can be set to smaller values compared with the case of heavier nuclei. For stable *sd* shell nuclei, the pairing correlations are expected to play only a minor role, which might justify our calculation of ^{24}Mg without pairing to some extent.

As for the cutoffs, we adopt the well-known energy cutoff [4] rather than simple cutoff by major shell numbers. All basis states below $E = (m_x + 0.5)\omega_x + (m_y + 0.5)\omega_y + (m_z + 0.5)\omega_z \leq 9.5\omega_0$ (MeV) are taken for the expansion of nucleon and meson fields (as stated in Ref. 2, twice values of $\omega_{x,y,z}$ and ω_0 are used for the meson fields). Deformation parameters of the harmonic oscillator basis are fixed to $\beta_0 = 0.5$ and $\gamma = 0$.

Figure 1 shows the total spin of the ground state band in ^{24}Mg. Although the spin value corresponding to the band termination is 12, the result is shown only up to $\Omega = 2.5$ MeV ($I \sim 8$). At $I = 8$, the $\gamma \sim -120°$ non-collective state is energetically favored and $I = 8$ state which belongs to the ground state band does not become the yrast. The moment of inertia which is shown in Fig. 2 increases slightly with rotational frequency. The quadrupole moment decreases from $Q \sim 107\,\text{fm}^2$ (at $\Omega = 0.1\,\text{MeV}$) to 83 fm^2 (at $\Omega = 2.5\,\text{MeV}$). The triaxial deformation is rather small at all frequencies considered.

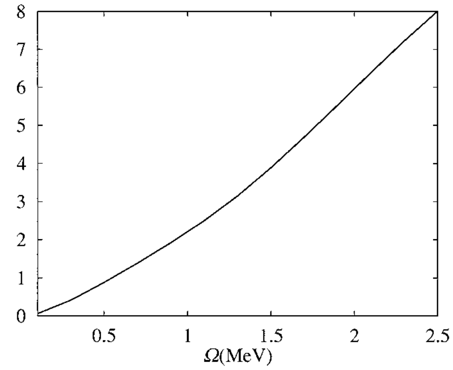

Fig. 1. Total spin of the ground state rotational band in ^{24}Mg as a function of the rotational frequency.

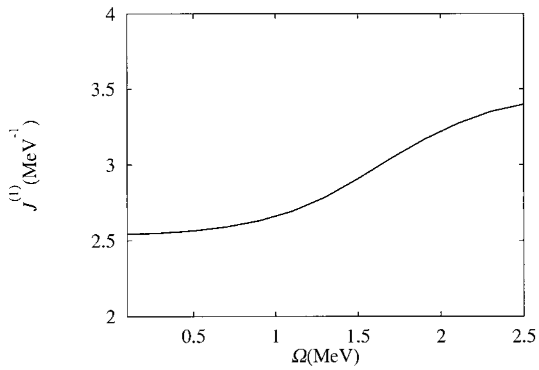

Fig. 2. Kinematical moment of inertia of the ground state rotational band in ^{24}Mg as a function of the rotational frequency.

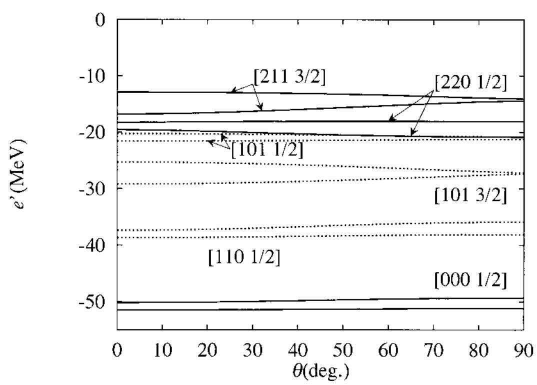

Fig. 3. Single neutron routhians of the ground state rotational band in ^{24}Mg as functions of the tilted angle. All orbits below Fermi level are shown. Solid lines show the $\pi = +$ orbits while dotted lines correspond to the $\pi = -$ ones.

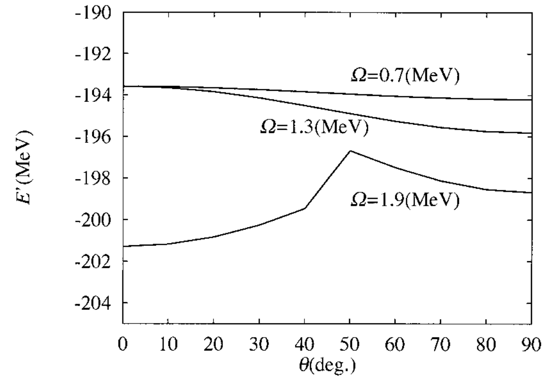

Fig. 4. Total routhians of the ground state rotational band in ^{24}Mg as functions of the tilted angle.

Now we turn to the tilted axis cranking calculation. Figure 3 shows all the single neutron routhians below Fermi level as functions of the tilted angle θ at $\Omega = 1.3\,\mathrm{MeV}$. All orbits seem to favor the PAC state (ether $\theta = 90^\circ$ or 0°) although some of them are rather flat. In Fig. 4, the total routhians are shown as functions of the tilted angle for selected values of Ω. Figure. 4 simply represents that the PAC states are in the lowest. At $\Omega = 0.7$ and $1.3\,\mathrm{MeV}$, $\theta = 90^\circ$ is favored, while at $\Omega = 1.9\,\mathrm{MeV}$ we find that the minimum is shifted to $\theta = 0^\circ$ non-collective state. As for this band, we could not find any tilted minima as expected.

Summary

We developed a numerical code of the RMF model which can treat the tilted axis rotation. The formulation is very similar to the 1-dimensional case, the only difference is now the metric tensor takes 3-dimensional form. As for the numerical technique, we again adopt the same technique as the 1-dimensional code. To check the code, we first calculated the ground state rotational band in ^{24}Mg. As expected, no tilted minima are observed in this nucleus. In the present version of our code, the pairing interactions are not included. We are now trying to include them within Relativistic Hartree Bogoliubov framework.

References

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