Toward consistent relativistic description of pairing in infinite matter and finite nuclei

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We construct relativistic effective particle-particle channel interactions which suit the gap equation for infinite nuclear matter. This is done by introducing a density-independent momentum-cutoff parameter to the relativistic mean field model so as to reproduce the pairing properties obtained by the Bonn-B potential and not to change the saturation property. The significance of the short-range correlation in the gap equation is also discussed.

$^1S_0$ pairing gap Δ in infinite nuclear matter is obtained by solving the gap equation,

$$\Delta(p) = -\frac{1}{8\pi^2}\int_0^\infty \bar{v}(p,k)\frac{\Delta(k)}{\sqrt{(E_k - E_{k_{\rm F}})^2 + \Delta^2(k)}}k^2 dk\,, \tag{1}$$

with $\bar{v}$ indicating the antisymmetrized matrix elements of the S wave particle-particle interaction $v_{\rm pp}$. A wide-range integration up to high momenta is inevitable in strongly-coupled systems such as the nuclear many-body ones. One can see from this equation that the physical ingredients are the single-particle energies E_k and $v_{\rm pp}$, and therefore, various theoretical approaches can be classified according to them irrespective of whether non-relativistic or relativistic models are used. The first type is the full Hartree-Bogoliubov (HB) or Hartree-Fock-Bogoliubov (HFB) calculations which adopt common effective interactions to the particle-hole (p-h) and the particle-particle (p-p) channels. This is thought desirable in connection with the studies of finite-density systems such as heavy nuclei. The second one is those which adopt different interactions in the p-h and the p-p channels; the single-particle states are determined by a HB or an HFB calculation using an effective interaction while the pairing properties are calculated with another one. This can be regarded as a practical alternative in nuclear structure study. The third one is those which adopt the single-particle states obtained by a Brueckner-Hartree-Fock (BHF) calculation and the corresponding *bare* interaction in the gap equation. This is regarded as the best way at tree level from a microscopic viewpoint; the use of the G-matrices in the p-p channel is known to give larger gaps. The polarization diagrams should be taken into account at the next order.

In contrast to the forty-year history of the non-relativistic study of superfluidity in infinite nuclear matter,[1)] the relativistic one has only a short history. The first study was done by Kucharek and Ring in 1991.[2)] This can be categorized to the first type according to the classification above. They adopted in the gap equation a one-boson exchange (OBE) interaction with the coupling constants of the relativistic mean field (RMF) model, which succeeded in reproducing the bulk properties of the finite-density nuclear many-body systems; both infinite matter and finite nuclei. But the resulting pairing gaps were about three times larger than those accepted in the non-relativistic studies. After a five-year blank, some attempts which are classified also into the first type above were done. But their results were insufficient and further investigations are definitely necessary. An attempt which is classified into the second type above was done by Rummel and Ring.[3)] They adopted the Bonn potential as $v_{\rm pp}$ whereas the single-particle states were still those from the RMF. Their results were very similar to those of a sophisticated approach adopting the single-particle states obtained by a Dirac-BHF (DBHF) calculation based on the Bonn potential, done by Elgarøy *et al.*,[4)] which is classified into the third type above. The result that these two kinds of calculations with the same $v_{\rm pp}$ gave similar pairing gaps is quite understandable if one roughly regards the RMF as simulating the single-particle states of the DBHF, and therefore, indicates that Rummel and Ring's RMF + Bonn calculation is realistic.

Then, by modeling the pairing properties given by the RMF + Bonn calculation, here we construct an effective p-p channel interaction based on the OBE with the coupling constants of the RMF and therefore keeping the spirit of the full HB method. In other words, we aim at constructing a relativistic effective p-p interaction which can play a role similar to that of the Gogny force in the non-relativistic studies which is thought to resemble a bare interaction[5)] in the p-p channel. However, one thing we have to bear in mind is the double counting problem of the short-range correlation. We will come back to this issue later.

Our policy of constructing such an effective interaction is to introduce a density-independent parameter Λ so as not to change the Hartree part with the momentum transfer $\mathbf{q} = 0$ which determines single-particle states, respecting that the original parameters of the RMF are density-independent. Since the high-momentum part of the interaction in the RMF does not have a firm experimental basis, we suppose there is room to modify that part. Two actual ways of the modification are examined: One is a sudden cutoff; the upper bound of the momentum integration in the gap equation (Eq. (1)) and the effective mass equation (Eq. (7)) is cut at Λ while $v_{\rm pp}$ is left unchanged. The other is a smooth cutoff; a form factor $f(\mathbf{q}^2)$, $\mathbf{q} = \mathbf{p} - \mathbf{k}$, containing Λ is applied to each nucleon-meson vertex in $v_{\rm pp} = \bar{v}(p,k)$ while the upper bound of the integrals is conceptually infinity, which is replaced by a number large enough, $20\,{\rm fm}^{-1}$ in the present study, numerically.

Since there is no decisive reasoning to choose a specific form, we examine three types,

$$
\begin{aligned}
&\text{monopole:} \quad f(\mathbf{q}^2) = \frac{\Lambda^2}{\Lambda^2 + \mathbf{q}^2}\,, \\
&\text{dipole:} \quad f(\mathbf{q}^2) = \left(\frac{\Lambda^2}{\Lambda^2 + \mathbf{q}^2}\right)^2, \\
&\text{strong:} \quad f(\mathbf{q}^2) = \frac{\Lambda^2 - \mathbf{q}^2}{\Lambda^2 + \mathbf{q}^2}\,.
\end{aligned} \tag{2}
$$

All of them, including the sudden cutoff case, contain a parameter Λ.

The parameter Λ is determined so as to minimize the difference in the pairing properties from the results of the RMF + Bonn calculation. Here we adopt the Bonn-B potential because this has a moderate property among the available (charge-independent) versions A, B, and C. The pair wave function

$$\phi(k) = \frac{1}{2}\frac{\Delta(k)}{\sqrt{(E_k - E_{k_{\rm F}})^2 + \Delta^2(k)}} = \frac{1}{2}\frac{\Delta(k)}{E_{\rm qp}(k)} = u_k v_k \tag{3}$$

is related to the gap at the Fermi surface

$$\Delta(k_{\rm F}) = -\frac{1}{4\pi^2}\int_0^\infty \bar{v}(k_{\rm F}, k)\phi(k)k^2 dk\,, \tag{4}$$

and its derivative determines the coherence length

$$\xi = \left(\frac{\int_0^\infty |\frac{d\phi}{dk}|^2 k^2 dk}{\int_0^\infty |\phi|^2 k^2 dk}\right)^{\frac{1}{2}}, \tag{5}$$

which measures the spatial size of the Cooper pairs. These expressions indicate that $\Delta(k_{\rm F})$ and ξ carry independent information, ϕ and $\frac{d\phi}{dk}$, respectively, in strongly-coupled systems, whereas they are intimately related to each other in weakly-coupled ones as those often treated in condensed-matter physics. Therefore we search for Λ which minimizes

$$\chi^2 = \frac{1}{2N}\sum_{k_{\rm F}}\left\{\left(\frac{\Delta(k_{\rm F})_{\rm RMF} - \Delta(k_{\rm F})_{\rm Bonn}}{\Delta(k_{\rm F})_{\rm Bonn}}\right)^2 + \left(\frac{\xi_{\rm RMF} - \xi_{\rm Bonn}}{\xi_{\rm Bonn}}\right)^2\right\}. \tag{6}$$

The actual numerical task is to solve the gap equation (Eq. (1)) and the effective mass equation for the nucleon,

$$M^* = M - \frac{g_\sigma^2}{m_\sigma^2}\frac{\gamma}{2\pi^2}\int_0^\infty \frac{M^*}{\sqrt{k^2 + M^{*\,2}}} v_k^2 k^2 dk\,, \tag{7}$$

where the upper bound of the integrals is replaced by Λ in the sudden cutoff case. The spin-isospin factor $\gamma = 4$ and 2 indicate symmetric nuclear matter and pure neutron matter, respectively. These equations couple to each other through

$$
\begin{aligned}
v_k^2 &= \frac{1}{2}\left(1 - \frac{E_k - E_{k_{\rm F}}}{\sqrt{(E_k - E_{k_{\rm F}})^2 + \Delta^2(k)}}\right), \\
E_k &= \sqrt{k^2 + M^{*\,2}} + g_\omega \langle\omega^0\rangle\,,
\end{aligned} \tag{8}
$$

where $\langle\omega^0\rangle$ is the expectation value of the time component of the vector meson field. The adopted Lagrangian is the standard σ-ω model with $g_\sigma^2 = 91.64$, $g_\omega^2 = 136.2$, $m_\sigma = 550$ MeV, $m_\omega = 783$ MeV, and $M = 939$ MeV. N in χ^2 is taken to be 11; $k_{\rm F} = 0.2, 0.3, \ldots, 1.2\,{\rm fm}^{-1}$. In the following, the results for symmetric nuclear matter are presented. Those for pure neutron matter are very similar except that $\Delta(k_{\rm F})$ is a little larger due to a larger effective mass M^* as shown in Fig. 1 (a).

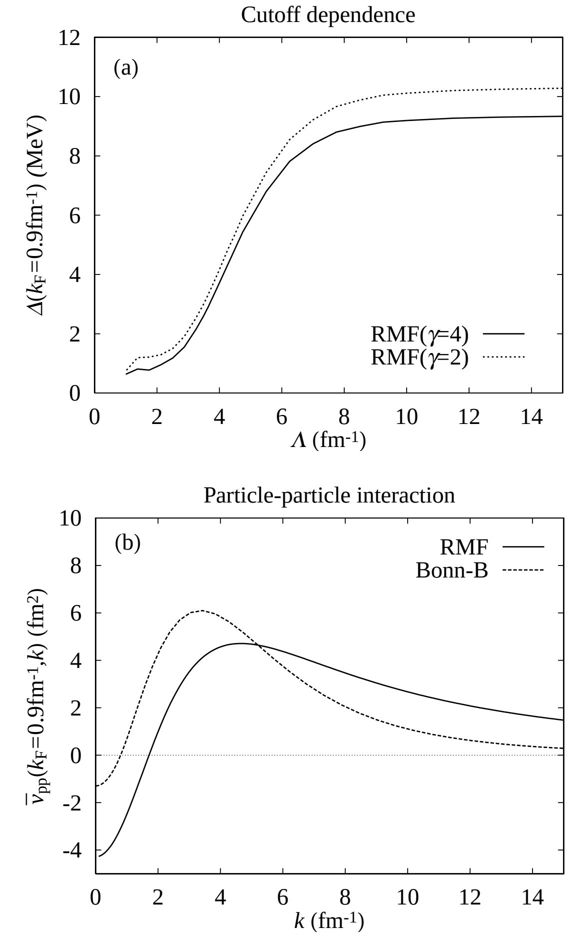

Fig. 1. (a): Pairing gap at the Fermi surface, $k_{\rm F} = 0.9\,{\rm fm}^{-1}$, obtained by the relativistic mean field model, as functions of the cutoff parameter Λ in the numerical integrations. $\gamma = 4$ and 2 represent the results for symmetric nuclear matter and pure neutron matter, respectively. (b): Matrix element $\bar{v}(k_{\rm F}, k)$ as functions of the momentum k, with a Fermi momentum $k_{\rm F} = 0.9\,{\rm fm}^{-1}$.[6)]

First we present the results of the sudden cutoff. Figure 1 (a) shows that the Λ-dependence of $\Delta(k_{\rm F})$ at $k_{\rm F} = 0.9\,{\rm fm}^{-1}$, where it becomes almost maximum. The dependence is strong around 3–8 ${\rm fm}^{-1}$ and saturation is reached around 10 ${\rm fm}^{-1}$. One can see from this it is possible to replace the infinity in the upper bound of the momentum integrations in the smooth cutoff cases with 20 ${\rm fm}^{-1}$ with enough accuracy. An important point is that the repulsive part also gives positive contributions to $\Delta(k_{\rm F})$ because $\Delta(k)$ changes the sign slightly after $\bar{v}(k_{\rm F}, k)$ does as seen in Fig. 4 later. Accordingly another plateau is seen around 2 ${\rm fm}^{-1}$. This corresponds to the zero in $v_{\rm pp}$ shown in Fig. 1 (b). The repulsive part at momenta higher than this is often abandoned but it is important to take into account this part correctly for obtaining physical gaps.

Figure 2 (a) shows the curvatures of χ^2 with respect to Λ both of the sudden cutoff and of the three types of form factor. Their steepness reflects the strength of the Λ-dependence shown in Fig. 2 (b); the dependence is rather mild in the smooth cutoff (form factor) cases. From this we chose 3.60 fm^{-1} for the sudden cutoff, 7.26, 10.66, and 10.98 fm^{-1} for the three types of form factor, respectively. Cutoff parameters with similar magnitudes are also suggested in the studies of medium-energy heavy-ion collisions.[7)]

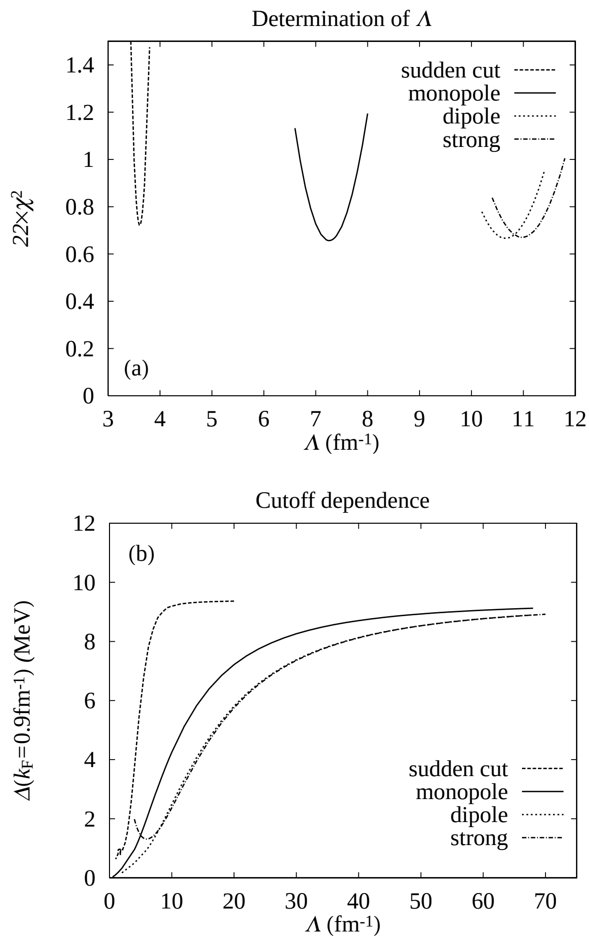

Fig. 2. (a): Curvature of χ^2 in Eq. (6) with respect to the cutoff parameter $\mathbf{\Lambda}$. (b): Large scale $\mathbf{\Lambda}$-dependence of the pairing gap at the Fermi surface, $k_\mathrm{F} = 0.9\ \mathrm{fm}^{-1}$. Note that the scale of the abscissa is different.

Figure 3 presents the results for $\Delta(k_\mathrm{F})$ and ξ as functions of k_F, obtained by using the cutoff parameters determined above. All the four cases examined, including the sudden cutoff, reproduce the results from the Bonn-B potential very well in wide and physically relevant density range, in the sense that pairing in finite nuclei occurs near the nuclear surface where the density is lower than the saturation point and that the calculated range of k_F almost corresponds to that of the inner crust of neutron stars. This is our first conclusion. The overall slight peak shift to higher k_F in $\Delta(k_\mathrm{F})$ and the deviation in ξ at the highest k_F are brought about by the systematic deviation in the critical density where the gap closes, between the calculations adopting bare v_pp and those adopting effective ones. It has not been studied enough that which is more realistic. The deviation at the lowest k_F is due to the feature that the present model is based on the mean-field picture for the finite-density systems. Actually, in such an extremely dilute system, the effective-range approximation for free scattering holds well.

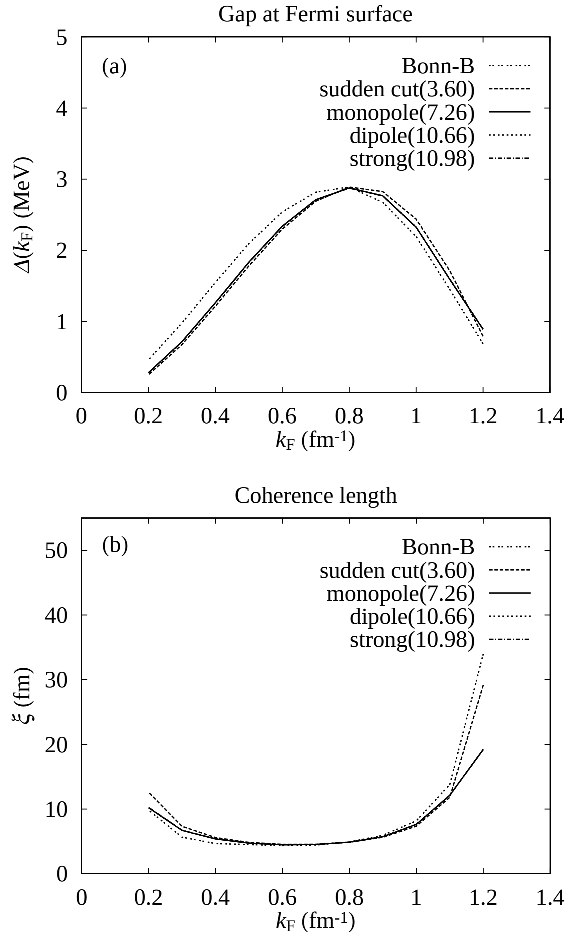

Fig. 3. (a): Pairing gap at the Fermi surface, and (b): coherence length, as functions of the Fermi momentum k_F, obtained by adopting the Bonn-B potential and the four kinds of effective interaction constructed in this study.

Next we look into the k-dependence at $k_\mathrm{F} = 0.9\,\mathrm{fm}^{-1}$. Figure 4 (a) shows the pair wave function. It is evident that all the four cases give the result identical to the Bonn-potential case within the width of the lines. This demonstrates clearly the effectiveness of the interactions constructed here as v_pp in the gap equation. This quantity peaks at $k = k_\mathrm{F}$ as seen from Eq. (3). The width of the peak represents the reciprocal of the coherence length, and its asymmetric shape indicates strongly-coupled feature. Equation (3) shows that $\phi(k)$ is composed of $\Delta(k)$ and the quasiparticle energy $E_\mathrm{qp}(k)$. Figure 4 (b) graphs the former. The gaps of all the five cases are almost identical up to $k \sim 2k_\mathrm{F}$, and deviations are seen only at higher momenta where $E_\mathrm{qp}(k)$ are large and accordingly pairing is not important. This is not a trivial result since the bare interaction is more repulsive than the effective ones constructed here even at the momentum region where $\Delta(k)$ are almost identical as shown in Fig. 4 (c).

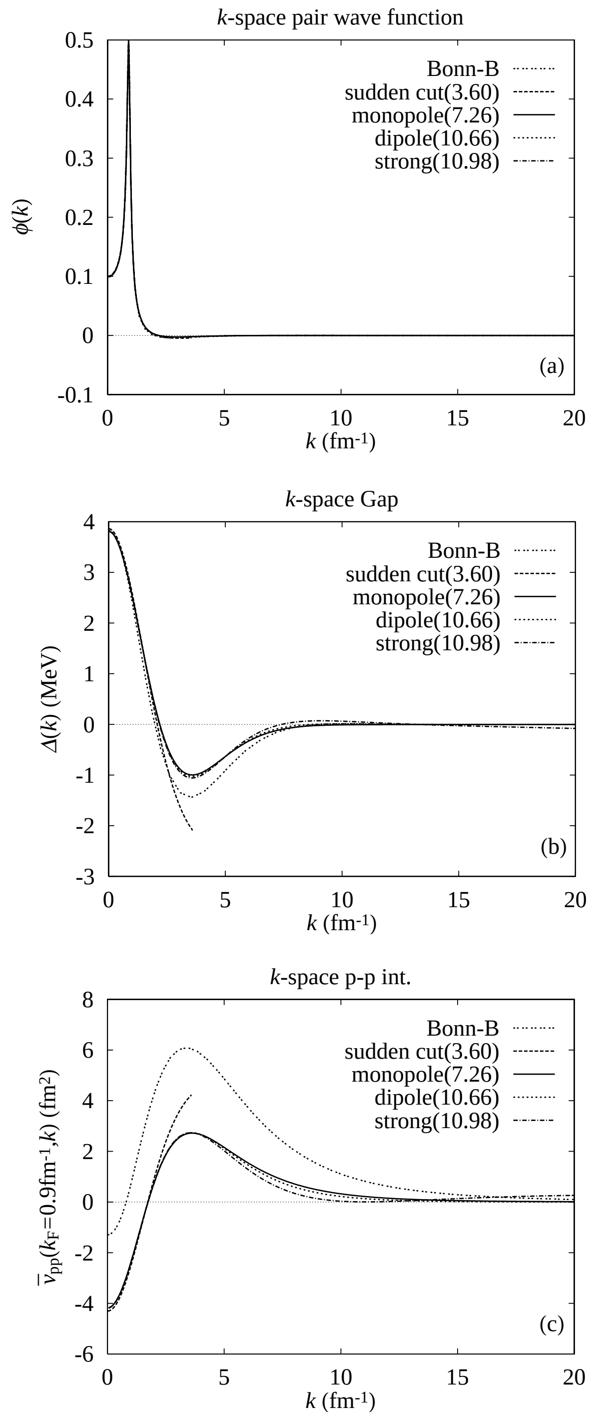

Fig. 4. (a): Pair wave function, (b): pairing gap, and (c): matrix element $\bar{v}(k_{\mathrm{F}}, k)$, as functions of the momentum k, calculated at a Fermi momentum $k_{\mathrm{F}} = 0.9\,\mathrm{fm}^{-1}$, by adopting the Bonn-B potential and the four kinds of effective interaction constructed in this study.

Finally we turn to r space in order to look into the physical contents further. Since the original form of the gap equation (Eq. (1)) can be rewritten as a convolution in the non-relativistic limit,

$$\Delta(\mathbf{p}) = -\int \bar{v}(\mathbf{p}-\mathbf{k})\phi(\mathbf{k})\frac{d^3k}{(2\pi)^3}\,, \qquad (1')$$

this is Fourier-transformed to

$$\Delta(\mathbf{r}) = -\bar{v}(\mathbf{r})\phi(\mathbf{r}) \qquad (1'')$$

in r space. One can see from this expression that, assuming $\Delta(\mathbf{r})$ is finite, $\phi(\mathbf{r})$ is pushed outwards when $\bar{v}(\mathbf{r})$ has a repulsive core, as the Brueckner wave functions.[8)] This is the reason why the use of the G-matrices in the gap equation is said to cause the double counting of the short-range correlation. The calculated r-space pair wave functions at $k_{\mathrm{F}} = 0.9\,\mathrm{fm}^{-1}$ are shown in Fig. 5 (a). Appreciable differences are seen only in the core region. The coherence length, that is a typical spatial scale of pairing correlation, is about 6 fm at this k_{F} as shown in Fig. 3 (b); this is almost one order of magnitude larger than the size of the core region. Therefore, practically we can safely use the effective interactions,

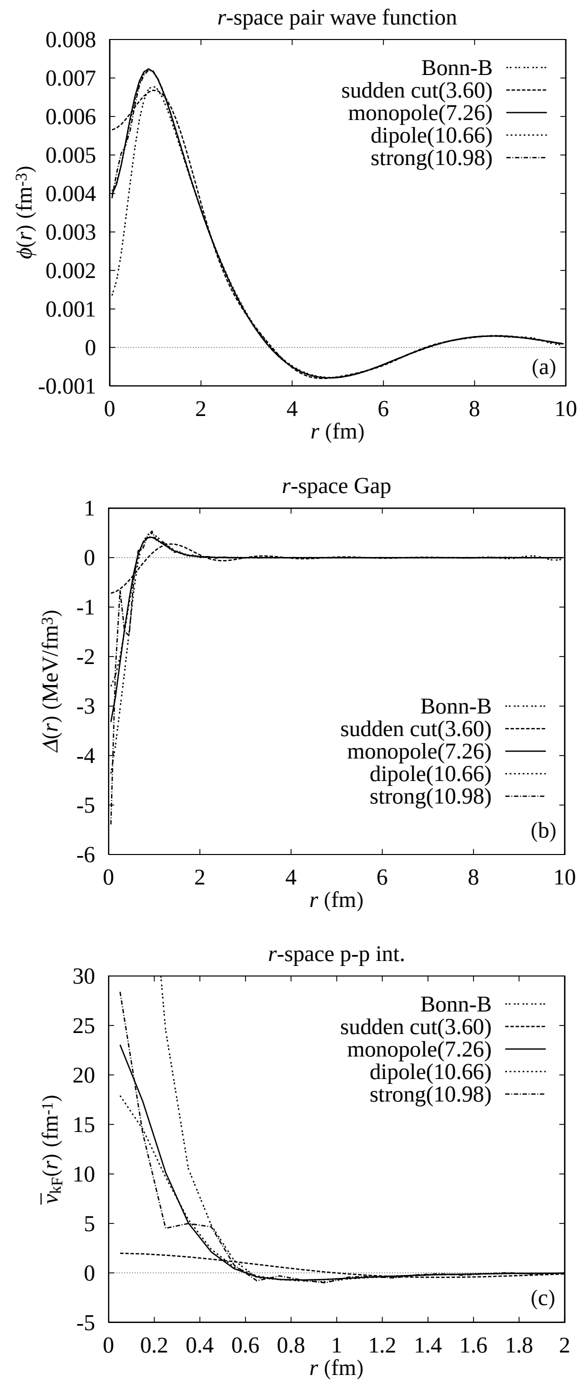

Fig. 5. The same as Fig. 4 but as functions of the distance r. Note that the scale of the abscissa in (c) is different from those in (a) and (b).

including the sudden cutoff one, constructed here for the gap equation.

Figure 5 (b) shows the corresponding $\Delta(r)$. The gaps are positive at the outside of the core and negative inside in all cases. Note here that the gap equation is invariant with respect to the overall sign inversion and we defined as $\Delta(k_{\rm F}) > 0$. Their depths at the inside region reflect the heights of the repulsive core in the Fourier transform of $\bar{v}(k_{\rm F}, k)$ shown in Fig. 5 (c). Note that shown are the Fourier transforms of $\bar{v}(p = k_{\rm F}, k)$ with respect to k, not those of $\bar{v}(q = |\mathbf{p} - \mathbf{k}|)$ with respect to q. They resemble each other at $k \gg k_{\rm F}$, and therefore at short distance. The patterns of oscillation also reflect the behavior of $\bar{v}$: The sudden cutoff case behaves somewhat differently from others due to the lack of high-momentum components. The additional staggering in the strong form factor case stems from $\bar{v}(p, k) = 0$ at $\Lambda = |\mathbf{q}| = |\mathbf{p} - \mathbf{k}|$; this gives an additional oscillatory structure in r space with a period $\sim \pi/\Lambda$. In this sense, the monopole and the dipole ones are the best whereas two others are also practically usable.

These analyses prove that some p-p interactions with strongly different short-range behavior can give almost identical pairing properties. This is another aspect of the short-range correlation in the gap equation. Note that the difference at short distance is reflected in a wide region in k space as shown in Fig. 4 (c). Therefore the character of the interactions constructed here based on the RMF is not only to simulate the G-matrices in the DBHF calculations in the sense that the $\mathbf{q} = 0$ Hartree part reproduces the saturation but also to give desired pairing properties as the Gogny force by improving the $\mathbf{q} \neq 0$ part.

To summarize, we have constructed relativistic effective particle-particle channel interactions which suit the gap equation for infinite nuclear matter based on the RMF. This has been accomplished by introducing a density-independent momentum-cutoff parameter to the standard RMF so as to reproduce the pairing properties obtained by adopting the Bonn-B potential and not to change the saturation property. Four kinds of parameterization were examined. All of them give practically identical results. Among them the monopole and the dipole ones have the best properties. This investigation has also clarified that some interactions with strongly different short-range behavior can give practically identical pairing properties. This is another aspect of the short-range correlation in the gap equation.

Now we are ready to study the polarization effects which are known to be important in the non-relativistic calculations beyond tree level. In particular, the behavior of the gap near the saturation density is to be studied. On the other hand, in order to extend the present study to asymmetric matter and finite nuclei, it is important to take into account isovector mesons and the non-linear self-coupling terms which are known to be crucial for describing finite nuclei quantitatively. These will be studied in forthcoming papers.

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