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Constructing Effective Pair Wave Function from Relativistic Mean Field Theory with a Cutoff

Tomonori TANIGAWA*) and Masayuki MATSUZAKI*,**)

Department of Physics, Kyushu University, Fukuoka 812-8581, Japan
**Department of Physics, Fukuoka University of Education Munakata 811-4192, Japan*

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We propose a simple method to reproduce the $^1S_0$ pairing properties of nuclear matter, which are obtained using a sophisticated model, by introducing a density-independent cutoff into the relativistic mean field model. This can be applied successfully to the physically relevant density range.

The $^1S_0$ pairing gap Δ in infinite nuclear matter is obtained by solving the gap equation,

$$\Delta(p) = -\frac{1}{8\pi^2}\int_0^\infty \bar{v}(p,k)\frac{\Delta(k)}{\sqrt{(E_k - E_{k_\mathrm{F}})^2 + \Delta^2(k)}}k^2 dk\,, \tag{1}$$

with $\bar{v}$ indicating the antisymmetrized matrix elements of the particle-particle interaction v_{pp}. One can see from this equation that the physical ingredients are the single-particle energies E_k and v_{pp}. In sophisticated microscopic approaches, the E_k are obtained from Brueckner-Hartree-Fock calculations with bare N-N interactions, which are fitted to the phase shifts of the N-N scatterings in free space. As for v_{pp}, most calculations employ bare interactions, while some others employ medium-renormalized interactions, such as the G matrices. Approaches involving calculations of the former type are based on the view that the gap equation itself possesses a mechanism to evade strong short-range repulsions, and, accordingly, use of medium-renormalized interactions results in a double counting.[1)-4)] The forty-year history of non-relativistic studies of the pairing problem[5)] has shown that all the bare N-N interactions that are fitted to the phase shifts give almost identical pairing gaps for the $^1S_0$ channel. This is because a separable approximation[6)] can be made for the S wave channels in which a virtual ($^1S_0$) or real ($^3S_1$) bound-state pole exists in the T matrices,[7)] and this leads to an approximate relation between the pairing gap and the phase shift applicable to the low-density region.[8)] Medium renormalizations are understood to cause the gap to become larger because they weaken the short-range repulsion. Irrespective of whether the medium renormalizations are included, the particle-hole polarizations should be considered in the next order according to a diagrammatic analysis of the gap equation,[9)] and it is said that they act to reduce the gaps.[10)-12)]

*) E-mail: tomo2scp@mbox.nc.kyushu-u.ac.jp
**) E-mail: matsuza@fukuoka-edu.ac.jp

Another approach to the pairing in nuclear matter is based on the effective interactions that are constructed from the beginning to describe finite-density systems. An example is the Gogny force,[13] which describes the bulk and the pairing properties of infinite matter quite well without any cutoffs,[14] and another is represented by various versions of the Skyrme forces, which require cutoffs for the description of the pairing.[15] From the viewpoint of the double counting of the short-range correlation mentioned above, however, the adequateness of the use of effective forces in the particle-particle (p-p) channel is not evident.[4] Although this is still an open problem, the Gogny force is said to act as a bare force in the p-p channel.[16]

Similarly to the studies discussed above, the first relativistic study of the pairing in nuclear matter was carried out in 1991[17] by adopting a phenomenological interaction, the relativistic mean field (RMF) model, which succeeded in reproducing the bulk properties of the finite-density nuclear many-body systems. But the resulting pairing gaps were about three times larger than those accepted as standard in the non-relativistic studies. After a five-year blank, various attempts to improve this result have begun. These attempts can be classified into two groups: The first one employs $v_{\rm pp}$ which are consistent with the particle-hole (p-h) channel, i.e. the single-particle states,[18-21] and the second one employs $v_{\rm pp}$ which are not explicitly consistent with the p-h channel.[22,23] In addition to these works which are based on the single-particle states of the RMF model, there exists another[24] which is based on the single-particle states obtained through the Dirac-Brueckner-Hartree-Fock (DBHF) calculation.[25] We refer to this as the third type hereafter. The result that the calculations of the second and the third types give almost identical pairing gaps indicates that the pairing properties are determined predominantly by the choice of the p-p channel interaction, irrespective of the details of the single-particle states. In addition, the feature that the obtained gaps are very similar to those given by the non-relativistic calculations adopting bare interactions in the p-p channel supports this further. As for the first type, a more elaborate calculation, such as one including the N-$\bar{N}$ polarizations, would be necessary.[20] As a complement to this kind of study, however, simpler methods suitable for realistic applications are also desirable. Examples for which realistic pairing strengths are indispensable are studies of the crust matter in neutron stars and finite open-shell nuclei. In particular, aside from the practical successes of the "relativistic" Hartree-Fock-Bogoliubov (HFB) calculations implemented by a non-relativistic force,[26] tractable relativistic $v_{\rm pp}$ derived from the Lagrangian of the RMF model are needed to keep the concept of the HFB calculation.

The purpose of this paper is to construct a relativistic effective force which can be used also in the p-p channel as the Gogny force in the non-relativistic calculation. Therefore, first of all, we consider the difference between an effective force and a bare force. In Fig. 1(a) the one-boson exchange $v_{\rm pp}$ with the coupling constants of the σ-ω model, which is the simplest version of the RMF model, is shown in comparison with the Bonn-B potential,[25] which is an example of the relativistic bare N-N interactions. Their shapes differ greatly. This is because the former is constructed so as to reproduce the saturation property without the short-range correlations, while the latter reproduces it in the DBHF calculation which implies them. This leads to the characteristic feature of the former that both the small-momentum negative *off-diagonal* matrix elements and the large-momentum positive ones are stronger than those for the latter. Both of them enhance the pairing gap, as discussed below.

The momentum integration in Eq. (1) should run to infinity when bare N-N interactions are adopted. In contrast, there is room to introduce a momentum cutoff when we adopt some phenomenological interactions, which are meaningful only for small momenta, as the Skyrme force. Evidently the assumption of the RMF model that the nucleon is a point particle cannot be justified at sufficiently large momenta. Combining this fact with the strong cutoff dependence in the momentum region $3-8$ fm^{-1} in Fig. 1(b) suggests the possibility to choose a proper cutoff which describes the pairing gap quantitatively. Note that the necessity of cutting off the large -momentum repulsion in the $v_{\rm pp}$ derived from the RMF model has also been suggested in studies of medium-energy heavy-ion collisions.[27),28)] This is interesting in the respect that two different phenomena, which involve large-momentum transfers, suggest similar cutoffs in the RMF-based p-p interaction.

In order to describe superfluidity quantitatively, not only the pair wave function

$$\phi(k) = \frac{1}{2}\frac{\Delta(k)}{\sqrt{(E_k - E_{k_{\rm F}})^2 + \Delta^2(k)}}\,, \tag{2}$$

which determines the gap at the Fermi surface

$$\Delta(k_{\rm F}) = -\frac{1}{4\pi^2}\int_0^\infty \bar{v}(k_{\rm F},k)\phi(k)k^2dk\,, \tag{3}$$

but also its derivative, which determines the coherence length[29)]

$$\xi = \left(\frac{\int_0^\infty |\frac{d\phi}{dk}|^2k^2dk}{\int_0^\infty |\phi|^2k^2dk}\right)^{\frac{1}{2}}\,, \tag{4}$$

should be reproduced. The latter quantity measures the spatial size of the Cooper pair. In weakly-coupled systems, in which $\Delta(k_{\rm F})$ is determined by the diagonal matrix element $v(k_{\rm F},k_{\rm F})$ only and $\xi \gg d$ (where d is the interparticle distance), $\Delta(k_{\rm F})$ and ξ are intimately related to each other. But this

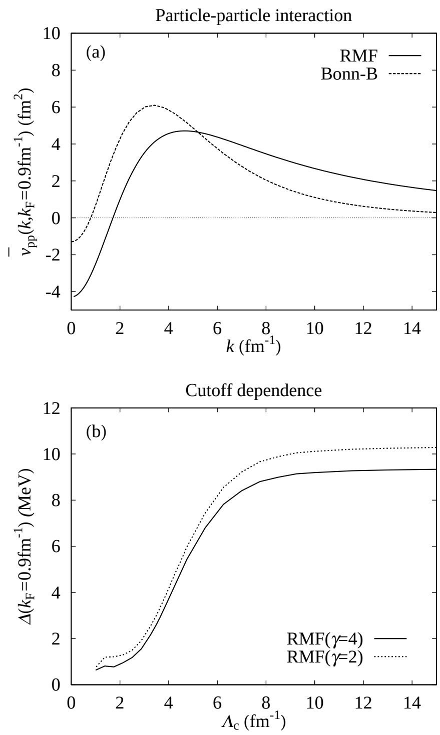

Fig. 1. (a) Matrix element $\bar{v}_{\rm pp}(k,k_{\rm F})$ as functions of the momentum k, with the Fermi momentum $k_{\rm F} =$ 0.9 fm^{-1}. The solid and dashed curves indicate the results obtained by the relativistic mean field model and the Bonn-B potential, respectively. (b) Pairing gap at the Fermi surface, $k_{\rm F} =$ 0.9 fm^{-1}, obtained from the relativistic mean field model, as functions of the cutoff parameter in the numerical integrations. The solid and dotted curves indicate the results for symmetric nuclear matter and pure neutron matter, respectively.

does not hold for nuclear many-body systems, and the off-diagonal matrix elements $v(k_{\rm F}, k)$ play important roles. Therefore here we attempt to find a density-independent cutoff $\Lambda_{\rm c}$ for the upper bound of the integrals in Eqs. (3) and (4) so as to reproduce, in a wide density range, $\Delta(k_{\rm F})$ and ξ obtained by adopting the Bonn-B potential. In other words, we attempt to introduce an extra parameter into the σ-ω model to fit the pairing properties described by a sophisticated model without changing the bulk properties.

The outline of the numerical calculations is as follows: We start from the σ-ω model with the no-sea approximation, as we confirmed in Ref. 21) that the Dirac sea effects were negligible. The parameters used are $M = 939$ MeV, $m_\sigma = 550$ MeV, $m_\omega = 783$ MeV, $g_\sigma^2 = 91.64$, and $g_\omega^2 = 136.2$.[30] The calculations were done for symmetric nuclear matter ($\gamma = 4$) and pure neutron matter ($\gamma = 2$). The pairing gap at each momentum is calculated by the gap equation (1) with $\Lambda_{\rm c}$ and the effective mass equation

$$M^* = M - \frac{g_\sigma^2}{m_\sigma^2}\frac{\gamma}{2\pi^2}\int_0^{\Lambda_{\rm c}} \frac{M^*}{\sqrt{k^2 + M^{*2}}} v_k^2 k^2 dk\,. \tag{5}$$

Equations (1) and (5) couple to each other through

$$v_k^2 = \frac{1}{2}\left(1 - \frac{E_k - E_{k_{\rm F}}}{\sqrt{(E_k - E_{k_{\rm F}})^2 + \Delta^2(k)}}\right),$$
$$E_k = \sqrt{k^2 + M^{*2}} + g_\omega \langle \omega^0 \rangle\,. \tag{6}$$

We search for the value of $\Lambda_{\rm c}$ that minimizes

$$\chi^2 = \frac{1}{2N}\sum_{k_{\rm F}}\left\{\left(\frac{\Delta(k_{\rm F})_{\rm RMF} - \Delta(k_{\rm F})_{\rm Bonn}}{\Delta(k_{\rm F})_{\rm Bonn}}\right)^2 + \left(\frac{\xi_{\rm RMF} - \xi_{\rm Bonn}}{\xi_{\rm Bonn}}\right)^2\right\}. \tag{7}$$

Here we assume equal weights for $\Delta(k_{\rm F})$ and ξ. The single-particle states are determined by the σ-ω model in both the "RMF" and the "Bonn" cases, as in Refs. 22) and 23). The summation with respect to $k_{\rm F}$ is taken as $k_{\rm F} = 0.2, 0.3, \cdots, 1.2$ fm^{-1}, i.e. $N = 11$, since we do not anticipate that the present method is applicable to the $k_{\rm F} \sim 0$ case, as discussed later.

We found that $\Lambda_{\rm c} = 3.60$ fm^{-1} minimizes χ^2 for $\gamma = 4$. This value indicates that not only the small-momentum part, where $v(k_{\rm F}, k) < 0$ and $\Delta(k) > 0$, but also the large-momentum part, where $v(k_{\rm F}, k) > 0$ and $\Delta(k) < 0$, contribute (see Fig. 1(a)) as pointed out in Refs. 22) and 23). The cutoff smaller than 2 fm^{-1} determined in Ref. 19) leads to cutting the "repulsive" part completely, and this corresponds to choosing the plateau around 2 fm^{-1} in Fig. 1(b), as proposed in Ref. 15) in the case of the Skyrme force. The present result does not agree with these previous ones. Reference 18) reports a result different from ours. As discussed in Ref. 21), there are two reasons for this difference. One reason is that they adopted coupling constants which reproduced the saturation in the Hartree-Fock approximation, not the Hartree (so-called MFT) approximation. This leads to larger pairing gaps.[31] The other reason is the difference in the evaluation of the Dirac sea effects.

Figures 2(a) and (b) show how well the σ-ω model with Λ_c chosen above reproduces $\Delta(k_F)$ and ξ, respectively, obtained with the Bonn-B potential for symmetric nuclear matter. One can see some deviations between the two models both near $k_F \sim 0.2$ fm^{-1} and $k_F \sim 1.2$ fm^{-1}. As for the former, it is quite reasonable that the present model based on the mean field picture for finite-density systems does not give a good fit. Actually, in such an extremely dilute system, the effective-range approximation for the free scattering is quite good.[8] As for the latter, the deviation results from the fact that the superfluid phase in the RMF model, as well as in the Gogny force case,[22] disappears in a nearly Λ_c-independent manner at somewhat larger k_F than in the Bonn potential case. This at the same time causes the overall peak shift of $\Delta(k_F)$ to larger k_F and makes ξ at large k_F small. We should note, however, that the critical density or k_F, where the pairing gap disappears, has not been fully discussed yet. The result for pure neutron matter is very similar, except that $\Delta(k_F)$ is somewhat larger as seen in Fig. 1(b), and the superfluid phase survives up to somewhat larger k_F, due to larger values of M^* than in the symmetric matter case. Consequently, the present method gives a good fit still for $k_F \sim 1.2$ fm^{-1}. The density of neutron matter in the inner crust of neutron stars corresponds to 0.2 fm^{-1} $\lesssim k_F \lesssim 1.3$ fm^{-1}.[5] Therefore the present simple method covers the greater part of this range. In finite nuclei, pairing occurs near the nuclear surface, where the density is lower than the saturation point. The present method gives a good description of this region.

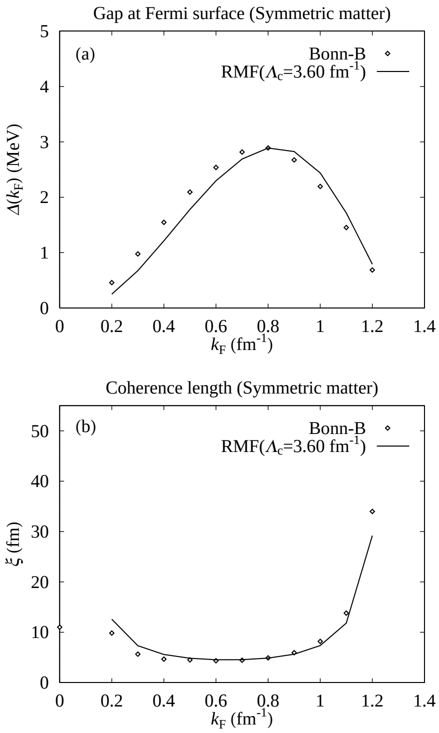

Fig. 2. Pairing gap at the Fermi surface (a), and coherence length (b) as functions of the Fermi momentum, calculated for symmetric nuclear matter. The solid curves and diamonds indicate the results obtained using the relativistic mean field model with a momentum cutoff $\Lambda_c = 3.60$ fm^{-1} and the Bonn-B potential, respectively.

To summarize, we proposed a method to reproduce the $^1S_0$ pairing properties of infinite nuclear matter, obtained using a sophisticated DBHF plus a full-range gap equation adopting the Bonn potential, by introducing a momentum cutoff into the gap equation with the relativistic mean field model. This method was shown to be applicable in a wide and physically relevant density range. This points the way to consistent (i.e., using the same interaction in the p-h and the p-p channels) relativistic HFB studies of neutron stars and finite nuclei. Finally, we remark that the cutoff we introduced is density-independent, since our approach is based on the density-independent — except the dependence through M^* — RMF interaction which was determined at the saturation point. This deserves further investigation.

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