

Dirac sea effects on superfluidity in nuclear matter

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We study two kinds of Dirac sea effects on the 1S_0 pairing gap in nuclear matter based on the relativistic Hartree approximation to quantum hadrodynamics and the Gor'kov formalism. We show that the vacuum fluctuation effect on the nucleon effective mass is more important than the direct coupling between the Fermi sea and the Dirac sea due to the pairing interaction. The effects of the high-momentum cutoff are also discussed. [S0556-2813(98)06712-0]

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Superfluidity caused by the pairing correlation between two nucleons with the linear or angular momenta opposite to each other is a key ingredient to describe quantitatively the thermal evolution of neutron stars and the structure of finite nuclei. As a way of description, relativistic models are attracting attention. The origin of relativistic nuclear models can be traced back to the work of Duerr [1]. Since Chin and Walecka succeeded in reproducing the saturation property of symmetric nuclear matter within the mean-field theory (MFT) with the no-sea approximation [2], quantum hadrodynamics (QHD) has described the bulk properties not only of infinite matter but of finite spherical, deformed, and rotating nuclei successfully [3,4]. These successes indicate that the particle-hole (p-h) interaction in QHD is realistic. However, various observables of nuclear many-body systems are sensitive to the single-particle properties around the Fermi surface. Knowledge of the residual particle-particle (p-p) interaction is required to describe them. Since this is still less understood, nonrelativistic interactions such as Gogny force are used in the pairing channel in practical "relativistic" MFT calculations.

The first study of the relativistic p-p interaction for the pairing channel in the nuclear medium was done by Kucharek and Ring [5]. They adopted, as the particle-particle interaction (v_{pp}) in the gap equation, a one-boson-exchange (OBE) interaction with ordinary relativistic MFT parameters which gave the saturation under the no-sea approximation. The resulting maximum gap was about 3 times larger than the accepted values in nonrelativistic calculations [6–11]. Various modifications to improve this result were proposed. They can be classified into two groups: One is to adopt the v_{pp} which is consistent with the p-h channel [12–14], and the other is to adopt effective ones which are not explicitly consistent with the p-h channel [15,3,16]. Among the former, we examined the p-h polarization in v_{pp} which reduced the pairing gap in the nonrelativistic models [17,18]. But the result was negative; this suggests that the nucleon-antinucleon ($N\bar{N}$) polarization should be taken into account simultaneously [14]. Before doing this, the antinucleon degrees of freedom have to be taken into account in the OBE step as done by Guimarães *et al.* [12]. In this paper, we discuss a more important effect of the Dirac sea than what they discussed.

Our formulation consists of three steps. In the first step, the equations of motion of the normal and the anomalous Green's functions G and F are derived. We start from the ordinary σ - ω model Lagrangian density

$$\begin{aligned}\mathcal{L} = & \bar{\psi}(i\gamma_\mu\partial^\mu - M)\psi + \frac{1}{2}(\partial_\mu\sigma)(\partial^\mu\sigma) - \frac{1}{2}m_\sigma^2\sigma^2 \\ & - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + g_\sigma\bar{\psi}\sigma\psi - g_\omega\bar{\psi}\gamma_\mu\omega^\mu\psi, \\ \Omega_{\mu\nu} = & \partial_\mu\omega_\nu - \partial_\nu\omega_\mu.\end{aligned}\quad (1)$$

The Hamiltonian density \mathcal{H} is derived from \mathcal{L} and the chemical potential μ is introduced in the Hamiltonian $H' = \int(\mathcal{H} - \mu\psi^\dagger\psi)d^3x$. The equations of motion for

$$\begin{aligned}G_{\alpha\beta}(x-x') = & -i\langle\tilde{0}|T\psi_\alpha(x)\bar{\psi}_\beta(x')|\tilde{0}\rangle, \\ F_{\alpha\beta}(x-x') = & -i\langle\tilde{0}|T\bar{\psi}_\alpha(x)\bar{\psi}_\beta(x')|\tilde{0}\rangle\end{aligned}\quad (2)$$

are derived from $i\partial_t\psi = [\psi, H']$. Here we note that the Green's functions have to be defined by the superfluid ground state $|\tilde{0}\rangle$ in order to introduce the antinucleon in the next step. Aside from this, this first step is essentially the same as the formulation of Ref. [5]. After evaluating the commutator with the interaction term in H' , the meson fields are eliminated by using the inverse of the Klein-Gordon equations. Here the gauge term in the propagator of the ω meson can be discarded since ω couples to the baryon current which is conserved on average in the present superfluid case. The result is

$$\begin{aligned}(\not{p} - M + \gamma^0\mu)_{\alpha\gamma}G_{\gamma\beta}(x-x') \\ = \delta_{\alpha\beta}\delta^4(x-x') + iV_{\alpha\gamma,\delta\epsilon} \\ \times \langle\tilde{0}|T\psi_\epsilon(t,\mathbf{y})\bar{\psi}_\delta(t,\mathbf{y})\psi_\gamma(x)\bar{\psi}_\beta(x')|\tilde{0}\rangle, \\ (-\not{\tilde{p}} - M + \gamma^0\mu)_{\gamma\alpha}F_{\gamma\beta}(x-x') \\ = iV_{\gamma\alpha,\delta\epsilon}\langle\tilde{0}|T\psi_\epsilon(t,\mathbf{y})\bar{\psi}_\delta(t,\mathbf{y})\bar{\psi}_\gamma(x)\bar{\psi}_\beta(x')|\tilde{0}\rangle, \\ V_{\alpha\gamma,\delta\epsilon} = -g_\sigma^2D_\sigma\delta_{\alpha\gamma}\delta_{\delta\epsilon} + g_\omega^2D_\omega(\gamma^\mu)_{\alpha\gamma}(\gamma_\mu)_{\delta\epsilon},\end{aligned}$$

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$$D_i = \frac{1}{-\Delta + m_i^2} \quad (i = \sigma, \omega), \quad (3)$$

with $x = (t, \mathbf{x})$ and $\tilde{\rho} = i\beta(\partial_t - \alpha \cdot \nabla)$. Here we introduced an instantaneous approximation since it was reported in the preceding works [12,13] that retardation effects are small. The time-ordered products on the right-hand side are decomposed in the manner of Gor'kov [19] by extending Wick's theorem,

$$\begin{aligned} & \langle \tilde{0} | T \psi_\epsilon(t, \mathbf{y}) \bar{\psi}_\delta(t, \mathbf{y}) \psi_\gamma(x) \bar{\psi}_\beta(x') | \tilde{0} \rangle \\ &= -\rho_{\epsilon\delta'}(0) \gamma_{\delta'\delta}^0 i G_{\gamma\beta}(x - x') + \rho_{\gamma\delta'}(\mathbf{x} - \mathbf{y}) \\ & \quad \times \gamma_{\delta'\delta}^0 i G_{\epsilon\beta}(y - x') - \kappa_{\gamma\epsilon}(\mathbf{x} - \mathbf{y}) i F_{\delta\beta}(y - x'), \\ & \langle \tilde{0} | T \psi_\epsilon(t, \mathbf{y}) \bar{\psi}_\delta(t, \mathbf{y}) \bar{\psi}_\gamma(x) \bar{\psi}_\beta(x') | \tilde{0} \rangle \\ &= -\rho_{\epsilon\delta'}(0) \gamma_{\delta'\delta}^0 i F_{\gamma\beta}(x - x') + \rho_{\epsilon\gamma'}(\mathbf{y} - \mathbf{x}) \gamma_{\gamma'\gamma}^0 \\ & \quad \times i F_{\delta\beta}(y - x') + \kappa_{\delta'\gamma'}^*(\mathbf{y} - \mathbf{x}) \gamma_{\delta'\delta}^0 \gamma_{\gamma'\gamma}^0 i G_{\epsilon\beta}(y - x'), \end{aligned} \quad (4)$$

where the normal and the anomalous densities are

$$\begin{aligned} \rho_{\alpha\beta}(\mathbf{x} - \mathbf{y}) &= \langle \tilde{0} | \psi_\beta^\dagger(t, \mathbf{y}) \psi_\alpha(x) | \tilde{0} \rangle, \\ \kappa_{\alpha\beta}(\mathbf{x} - \mathbf{y}) &= \langle \tilde{0} | \psi_\beta(t, \mathbf{y}) \psi_\alpha(x) | \tilde{0} \rangle. \end{aligned} \quad (5)$$

The self-energy and the pairing field are defined by these densities as

$$\Sigma_{\alpha\gamma}(\mathbf{x} - \mathbf{y}) = [V_{\alpha\gamma, \delta\epsilon} \rho_{\epsilon\delta'}(0) - V_{\alpha\epsilon, \delta\gamma} \rho_{\epsilon\delta'}(\mathbf{x} - \mathbf{y})] \gamma_{\delta'\delta}^0,$$

$$\Delta_{\alpha\gamma}(\mathbf{x} - \mathbf{y}) = \gamma_{\alpha\alpha'}^0 \gamma_{\gamma\gamma'}^0 V_{\alpha'\delta, \gamma'\epsilon} \kappa_{\delta\epsilon}(\mathbf{x} - \mathbf{y}). \quad (6)$$

The Fourier transform of the equations of motion for g and f defined by $G_{\gamma\beta} = g_{\gamma\beta'} \gamma_{\beta'\beta}^0$ and $F_{\gamma\beta} = f_{\gamma'\beta'} \gamma_{\gamma'\gamma}^0 \gamma_{\beta'\beta}^0$ is given by

$$\begin{pmatrix} (\omega - h + \mu)_{\alpha\gamma} & -\Delta_{\alpha\gamma} \\ \Delta_{\alpha\gamma}^* & (\omega + h^* - \mu)_{\alpha\gamma} \end{pmatrix} \begin{pmatrix} g_{\gamma\beta} \\ f_{\gamma\beta} \end{pmatrix} = \begin{pmatrix} \delta_{\alpha\beta} \\ 0 \end{pmatrix}, \quad (7)$$

where $h = \alpha \cdot \mathbf{k} + \beta[M + \Sigma(k)]$.

In the second step, we derive the equations for the Bogoliubov transformation amplitudes by expressing g and f in Eq. (7) in terms of them. We introduce the Dirac field in the Schrödinger picture,

$$\psi(\mathbf{x}) = \frac{1}{\sqrt{V}} \sum_{\lambda=(\mathbf{k}\sigma)} [a_\lambda U(\lambda) e^{i\mathbf{k}\cdot\mathbf{x}} + b_\lambda^\dagger V(\lambda) e^{-i\mathbf{k}\cdot\mathbf{x}}], \quad (8)$$

to manipulate the t dependence. Here the normalization of U and V is chosen to conform to Ref. [20]. The Bogoliubov amplitudes are defined by

$$\begin{aligned} A_\lambda &= \langle \tilde{0} | a_\lambda \eta_\lambda^\dagger | \tilde{0} \rangle, \quad B_\lambda = \langle \tilde{0} | b_{-\lambda}^\dagger \eta_\lambda^\dagger | \tilde{0} \rangle, \\ C_\lambda &= \langle \tilde{0} | a_{-\lambda}^\dagger \eta_\lambda^\dagger | \tilde{0} \rangle, \quad D_\lambda = \langle \tilde{0} | b_\lambda \eta_\lambda^\dagger | \tilde{0} \rangle, \end{aligned} \quad (9)$$

as the overlaps between the quasiparticle η_λ^\dagger ,

$$H' \eta_\lambda^\dagger | \tilde{0} \rangle = \mathcal{E}_\lambda \eta_\lambda^\dagger | \tilde{0} \rangle, \quad H' | \tilde{0} \rangle = 0, \quad (10)$$

and the nucleon or the antinucleon. These give explicit expressions for g and f :

$$\begin{aligned} g_{\gamma\beta}(\omega, \mathbf{k}) &= \frac{1}{\omega - \mathcal{E}_k + i\epsilon} \sum_s \{ |A_\lambda|^2 U_\gamma(\lambda) U_\beta^*(\lambda) + |B_\lambda|^2 V_\gamma(-\lambda) V_\beta^*(-\lambda) + A_\lambda B_\lambda^* U_\gamma(\lambda) V_\beta^*(-\lambda) \\ & \quad + B_\lambda A_\lambda^* V_\gamma(-\lambda) U_\beta^*(\lambda) \} + \frac{1}{\omega + \mathcal{E}_k - i\epsilon} \sum_s \{ |C_{-\lambda}|^2 U_\gamma(\lambda) U_\beta^*(\lambda) + |D_{-\lambda}|^2 V_\gamma(-\lambda) \\ & \quad \times V_\beta^*(-\lambda) + D_{-\lambda} C_{-\lambda}^* U_\gamma(\lambda) V_\beta^*(-\lambda) + C_{-\lambda} D_{-\lambda}^* V_\gamma(-\lambda) U_\beta^*(\lambda) \}, \\ f_{\gamma\beta}(\omega, \mathbf{k}) &= \frac{1}{\omega - \mathcal{E}_k + i\epsilon} \sum_s \{ C_\lambda A_\lambda^* U_\gamma^*(-\lambda) U_\beta^*(\lambda) + D_\lambda B_\lambda^* V_\gamma^*(\lambda) V_\beta^*(-\lambda) + C_\lambda B_\lambda^* U_\gamma^*(-\lambda) V_\beta^*(-\lambda) \\ & \quad + D_\lambda A_\lambda^* V_\gamma^*(\lambda) U_\beta^*(\lambda) \} + \frac{1}{\omega + \mathcal{E}_k - i\epsilon} \sum_s \{ C_{-\lambda} A_{-\lambda}^* U_\gamma^*(-\lambda) U_\beta^*(\lambda) + D_{-\lambda} B_{-\lambda}^* V_\gamma^*(\lambda) \\ & \quad \times V_\beta^*(-\lambda) + D_{-\lambda} A_{-\lambda}^* U_\gamma^*(-\lambda) V_\beta^*(-\lambda) + C_{-\lambda} B_{-\lambda}^* V_\gamma^*(\lambda) U_\beta^*(\lambda) \}. \end{aligned} \quad (11)$$

The former describes the normal propagation and the $\Delta N = 0$ pairing, and the latter describes the ordinary $\Delta N = 2$ pairing. This $\Delta N = 0$ pairing appears because Cooper pairs can be formed regardless of the sign of the single-nucleon energy. Substituting Eq. (11) into Eq. (7) and defining the matrix elements of the pairing field Δ as

$$\begin{aligned} \Delta(\lambda) &= e^{-i\alpha_+(\lambda)} U_\alpha^\dagger(\lambda) \Delta_{\alpha\gamma} T_{\gamma\beta} U_\beta(\lambda), \\ \tilde{\Delta}(\lambda) &= e^{-i\alpha_-(-\lambda)} V_\alpha^\dagger(-\lambda) \Delta_{\alpha\gamma} T_{\gamma\beta} V_\beta(-\lambda), \\ \delta(\lambda) &= e^{-i\alpha_-(-\lambda)} U_\alpha^\dagger(\lambda) \Delta_{\alpha\gamma} T_{\gamma\beta} V_\beta(-\lambda), \end{aligned} \quad (12)$$

for the Fermi sea pairing, the Dirac sea pairing, and the $\Delta N=0$ pairing, respectively, with the phase factors associated with time reversal [21],

$$TU(\lambda) = e^{i\alpha_+(\lambda)} U^*(-\lambda),$$

$$TV(-\lambda) = e^{i\alpha_-(-\lambda)} V^*(\lambda),$$

$$T = i\gamma^1 \gamma^3, \quad (13)$$

we obtain the equation of the Bogoliubov amplitudes:

$$\begin{pmatrix} \omega - E_k + \mu & 0 & -\Delta(\lambda) & -\delta(\lambda) \\ 0 & \omega + E_k + \mu & -\delta(\lambda) & -\tilde{\Delta}(\lambda) \\ -\Delta(\lambda) & -\delta(\lambda) & \omega + E_k - \mu & 0 \\ -\delta(\lambda) & -\tilde{\Delta}(\lambda) & 0 & \omega - E_k - \mu \end{pmatrix} \times \begin{pmatrix} A_\lambda \\ B_\lambda \\ C_\lambda \\ D_\lambda \end{pmatrix} = 0. \quad (14)$$

Here we used

$$h_{\alpha\gamma} U_\gamma(\lambda) = E_k U_\alpha(\lambda),$$

$$h_{\alpha\gamma} V_\gamma(-\lambda) = -E_k V_\alpha(-\lambda),$$

$$E_k = \sqrt{\mathbf{k}^2 + M^{\star 2}},$$

$$Th^* = hT, \quad T\Delta^* = \Delta T. \quad (15)$$

Note that we adopted notation such that Eq. (14) took the same form as that of Ref. [12] and chose a phase convention such that all the matrix elements were real. Among the eigenvalues,

$$\omega^2 = \left(E_k^2 + \mu^2 + \frac{1}{2}(\Delta^2 + \tilde{\Delta}^2) + \delta^2 \right) - \frac{1}{2} \sqrt{(4E_k\mu - \Delta^2 + \tilde{\Delta}^2)^2 + 4\delta^2[4E_k^2 + (\Delta + \tilde{\Delta})^2]} \quad (16)$$

corresponds to the Fermi sea pairing in the decoupling ($\delta \rightarrow 0$) limit.

In the third step, we express κ , and subsequently the matrix elements $\Delta(\lambda)$, $\tilde{\Delta}(\lambda)$, and $\delta(\lambda)$, in terms of $A_\lambda - D_\lambda$. The Fourier transform of κ in Eq. (5) is given by

$$\begin{aligned} \kappa_{\delta\epsilon}(\omega, \mathbf{k}) = 2\pi \delta(\omega + \mathcal{E}_k) \sum_s \{ & A_{-\lambda} C_{-\lambda} U_\epsilon(-\lambda) U_\delta(\lambda) + B_{-\lambda} D_{-\lambda} V_\epsilon(\lambda) V_\delta(-\lambda) \\ & + A_{-\lambda} D_{-\lambda} U_\epsilon(-\lambda) V_\delta(-\lambda) + B_{-\lambda} C_{-\lambda} V_\epsilon(\lambda) U_\delta(\lambda) \}. \end{aligned} \quad (17)$$

Substituting this into the Fourier transform of the second equation of Eqs. (6) gives

$$\begin{aligned} \Delta(\lambda) = & -\frac{i}{2} \int \frac{d^3p}{(2\pi)^3} \frac{1}{4E_k E_p} \left(\frac{-g_\sigma^2}{|\mathbf{k}-\mathbf{p}|^2 + m_\sigma^2} [\text{Tr}\{(\mathbf{k} + M^*)(\mathbf{p} + M^*)\} e^{i\alpha_+(\lambda)} A_\lambda C_\lambda \right. \\ & + \text{Tr}\{(\mathbf{k} + M^*)(\mathbf{p} - M^*)\} e^{i\alpha_-(-\lambda)} B_\lambda D_\lambda] + \frac{g_\omega^2}{|\mathbf{k}-\mathbf{p}|^2 + m_\omega^2} [\text{Tr}\{(\mathbf{k} + M^*)\gamma_\mu(\mathbf{p} + M^*)\gamma_\mu\} \\ & \times e^{i\alpha_+(\lambda)} A_\lambda C_\lambda + \text{Tr}\{(\mathbf{k} + M^*)\gamma_\mu(\mathbf{p} - M^*)\gamma_\mu\} e^{i\alpha_-(-\lambda)} B_\lambda D_\lambda] \Big), \\ \tilde{\Delta}(\lambda) = & -\frac{i}{2} \int \frac{d^3p}{(2\pi)^3} \frac{1}{4E_k E_p} \left(\frac{-g_\sigma^2}{|\mathbf{k}-\mathbf{p}|^2 + m_\sigma^2} [\text{Tr}\{(\mathbf{k} - M^*)(\mathbf{p} + M^*)\} e^{i\alpha_+(\lambda)} A_\lambda C_\lambda \right. \\ & + \text{Tr}\{(\mathbf{k} - M^*)(\mathbf{p} - M^*)\} e^{i\alpha_-(-\lambda)} B_\lambda D_\lambda] + \frac{g_\omega^2}{|\mathbf{k}-\mathbf{p}|^2 + m_\omega^2} [\text{Tr}\{(\mathbf{k} - M^*)\gamma_\mu(\mathbf{p} + M^*)\gamma_\mu\} \\ & \times e^{i\alpha_+(\lambda)} A_\lambda C_\lambda + \text{Tr}\{(\mathbf{k} - M^*)\gamma_\mu(\mathbf{p} - M^*)\gamma_\mu\} e^{i\alpha_-(-\lambda)} B_\lambda D_\lambda] \Big), \end{aligned}$$

$$\begin{aligned}
\delta(\lambda) = & -\frac{i}{2} \int \frac{d^3p}{(2\pi)^3} \frac{1}{4E_k E_p} \left(\frac{-g_\sigma^2}{|\mathbf{k}-\mathbf{p}|^2 + m_\sigma^2} [\text{Tr}\{\gamma^5(\mathbf{k}-M^*)\gamma^0\gamma^5(\mathbf{p}-M^*)\gamma^0\}e^{i\alpha_+(\lambda)}B_\lambda C_\lambda \right. \\
& - \text{Tr}\{\gamma^5(\mathbf{k}-M^*)\gamma^0\gamma^5(\mathbf{p}+M^*)\gamma^0\}e^{i\alpha_-(-\lambda)}A_\lambda D_\lambda] + \frac{g_\omega^2}{|\mathbf{k}-\mathbf{p}|^2 + m_\omega^2} [\text{Tr}\{\gamma^5(\mathbf{k}-M^*)\gamma^0\gamma_\mu\gamma^5(\mathbf{p}-M^*)\gamma^0\gamma_\mu\} \\
& \times e^{i\alpha_+(\lambda)}B_\lambda C_\lambda - \text{Tr}\{\gamma^5(\mathbf{k}-M^*)\gamma^0\gamma_\mu\gamma^5(\mathbf{p}+M^*)\gamma^0\gamma_\mu\}e^{i\alpha_-(-\lambda)}A_\lambda D_\lambda] \Big), \quad (18)
\end{aligned}$$

where both γ_μ in each line are covariant due to time reversal. Here we used $A_{-\lambda}D_{-\lambda} = -B_\lambda C_\lambda$, and so on, derived from the definition of κ , the s independence of $e^{i\alpha_+(\lambda)}A_\lambda C_\lambda$, and so on, and

$$\begin{aligned}
\sum_s U(\lambda)\bar{U}(\lambda) &= \frac{1}{2E_k}(\mathbf{k}+M^*), \\
\sum_s V(-\lambda)\bar{V}(-\lambda) &= \frac{1}{2E_k}(\mathbf{k}-M^*), \\
\sum_s U(\lambda)\bar{V}(-\lambda) &= -\frac{1}{2E_k}\gamma^5(\mathbf{k}-M^*)\gamma^0, \\
\sum_s V(-\lambda)\bar{U}(\lambda) &= \frac{1}{2E_k}\gamma^5(\mathbf{k}+M^*)\gamma^0, \\
\tilde{k} &= (E_k, -\mathbf{k}). \quad (19)
\end{aligned}$$

Equations (14) and (18) form a system of self-consistent equations. The nucleon effective mass M^* is determined by the self-consistent condition for the scalar part of the self-energy, the first equation of Eqs. (6),

$$\begin{aligned}
M^* = M - \frac{g_\sigma^2}{m_\sigma^2} \frac{\gamma}{2\pi^2} \int_0^{\Lambda_c} \frac{M^*}{\sqrt{\mathbf{p}^2 + M^{*2}}} C_p^2 p^2 dp \\
+ \frac{g_\sigma^2}{m_\sigma^2} \frac{1}{\pi^2} \left\{ M^{*3} \ln\left(\frac{M^*}{M}\right) - M^2(M^*-M) \right. \\
\left. - \frac{5}{2}M(M^*-M)^2 - \frac{11}{6}(M^*-M)^3 \right\}, \quad (20)
\end{aligned}$$

within the relativistic Hartree approximation (RHA) in which the divergence due to the Dirac sea is renormalized using the counterterms

$$\mathcal{L}_{\text{ct}} = \alpha_1 \sigma + \frac{1}{2!} \alpha_2 \sigma^2 + \frac{1}{3!} \alpha_3 \sigma^3 + \frac{1}{4!} \alpha_4 \sigma^4. \quad (21)$$

Hereafter we suppress the argument s , and therefore the direction of \mathbf{k} which defines the direction of s , of the gaps and the Bogoliubov amplitudes because they affect only the overall sign. The expression of the vacuum contribution in Eq. (20) is taken from the nonsuperfluid case [20]. We will discuss this later. Then the actual task is to solve the coupled equations (14), (18), and (20). If we neglect the vacuum fluctuation contribution in Eq. (20), the system of equations

corresponds to that of Ref. [12]. Here we note that we adopt the Hartree approximation with or without the Dirac sea contribution as in the no-sea study of Ref. [5] although the Fock terms are also included in Ref. [12].

Now we proceed to the numerical results. The parameters used are $M=939$ MeV, $m_\sigma=550$ MeV, $m_\omega=783$ MeV, and $g_\sigma^2=62.89$ for the RHA or 91.64 for the MFT, $g_\omega^2=79.78$ for the RHA or 136.2 for the MFT [20], and $\gamma=4$ (symmetric nuclear matter). First we look into the relative magnitudes of the gaps $\Delta(k)$, $\tilde{\Delta}(k)$, and $\delta(k)$. Since A_k and C_k are dominant among those associated with the eigenvalue, Eq. (16), $\Delta(k)$ and $\tilde{\Delta}(k)$ are much larger than $\delta(k)$. From the definition of κ in Eq. (5), two kinds of terms—one is antisymmetric in spin space and even with respect to the inversion of the momentum, and the other is symmetric and odd—are possible in $\kappa_{\delta\epsilon}(\omega, \mathbf{k})$. If finite-range effects are neglected, only six terms of the former type are possible as discussed in Ref. [13]. They correspond to scalar, pseudoscalar, and four-vector terms in κT . Among them, the scalar and the time component of the vector are dominant and consequently $\Delta(k)$ and $\tilde{\Delta}(k)$ are determined by these two terms as pointed out in Ref. [13]. Basically this applies also to the calculations of Ref. [12] and ours in which the terms of the latter type are also included. In contrast, $\delta(k)$ which measures the $\Delta N=0$ pairing is given by the pseudoscalar and pseudovector terms in κT . Since they contain $B_k C_k$ and $A_k D_k$, their typical values are the order of 10^{-5} MeV or less both in the RHA and in the MFT. Therefore, as for the magnitude of the $\Delta N=0$ coupling effect on $\Delta(k)$, our calculation does not agree with that of Ref. [12]. The origin of this disagreement is that their Eq. (54) does not have this structure, that different types of products of the Bogoliubov amplitudes appear in $\delta(k)$. The present result indicates that the Fermi sea pairing and the Dirac sea pairing decouple in accuracy of 10^{-6} . Accordingly, we use the expressions for $\delta(k)=0$ hereafter. Since the equations for A_k and C_k , decoupled from B_k and D_k , are equivalent to a gap equation

$$\Delta(k) = -\frac{1}{8\pi^2} \int_0^{\Lambda_c} \bar{v}_{\text{pp}}(k, p) \frac{\Delta(p)}{\sqrt{(E_p - E_{k_F})^2 + \Delta^2(p)}} p^2 dp, \quad (22)$$

the numerical task is greatly simplified to solving the coupled equations (20) and (22). Here $\bar{v}_{\text{pp}}(k, p)$ is an antisymmetrized matrix element of the adopted p-p interaction,

$$\bar{v}_{\text{pp}}(\mathbf{k}, \mathbf{p}) = \langle \mathbf{k}s', \tilde{\mathbf{k}}s' | V | \mathbf{p}s, \tilde{\mathbf{p}}s \rangle - \langle \mathbf{k}s', \tilde{\mathbf{k}}s' | V | \tilde{\mathbf{p}}s, \mathbf{p}s \rangle, \quad (23)$$

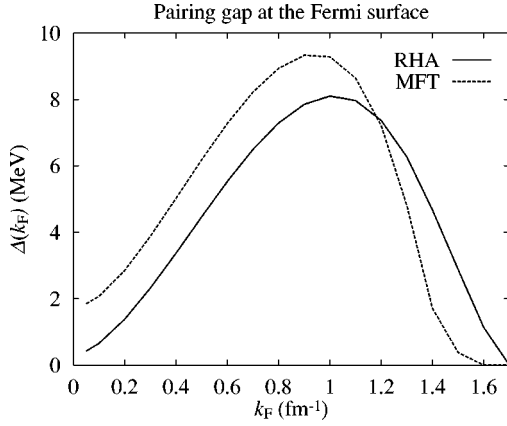


FIG. 1. Pairing gap in symmetric nuclear matter at the Fermi surface as functions of the Fermi momentum. Solid and dashed lines indicate the results obtained by taking and not taking into account the vacuum fluctuation contribution, respectively.

where V is the Fourier transform of the one in Eqs. (3), with an instantaneous approximation and an integration with respect to the angle between \mathbf{k} and \mathbf{p} to project out the S -wave component.

Next we compare the results of the RHA and the MFT obtained by adopting $\Lambda_c = 15 \text{ fm}^{-1}$ which is large enough for the numerical integrations to converge (see Fig. 3). These are presented in Fig. 1. This shows that the Dirac sea contributes to reducing the gap at low density while to enhancing it at high density. This can be understood as follows: Although both g_σ and g_ω are reduced in the RHA in comparison with the MFT, altogether they act to reduce the repulsion as shown in Fig. 2. Since both the low-momentum attraction and the high-momentum repulsion give positive contributions to $\Delta(k_F)$ as discussed in Refs. [15,16], the above-mentioned reduction of the repulsion leads to a reduction of $\Delta(k_F)$ in the RHA. Besides, the low-momentum attraction decreases steeply as the density increases especially in the MFT (not shown). This leads to a steeper reduction of $\Delta(k_F)$ at high density in the MFT than in the RHA. Here we note that this RHA calculation was done by assuming the same

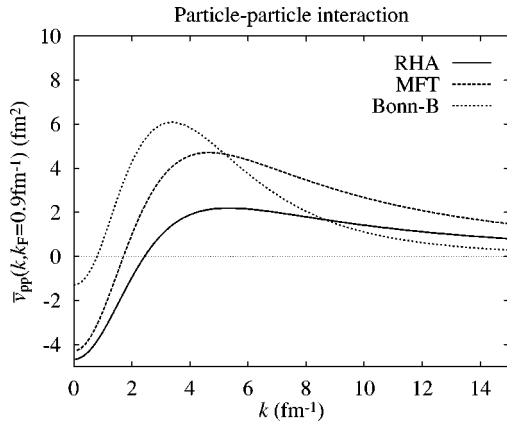


FIG. 2. Matrix element $\bar{v}_{pp}(k, k_F)$ as functions of the momentum k , with a Fermi momentum $k_F = 0.9 \text{ fm}^{-1}$. Solid and dashed lines indicate the results obtained by taking and not taking into account the vacuum fluctuation contribution, respectively. The Bonn-B potential for the 1S_0 channel is also shown by the dotted line.

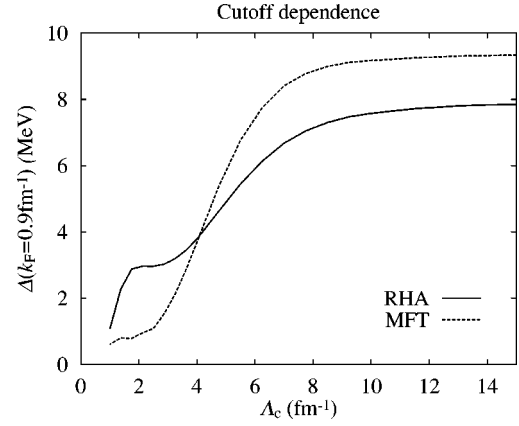


FIG. 3. Pairing gap at the Fermi surface, $k_F = 0.9 \text{ fm}^{-1}$, as functions of the cutoff parameter in the numerical integrations. Solid and dashed lines indicate the results obtained by taking and not taking into account the vacuum fluctuation contribution, respectively.

vacuum fluctuation contribution as the nonsuperfluid case as mentioned above. We believe that this approximation is practical because the effect of the pairing on M^* is negligible numerically except at very low density. This indicates that the bulk property M^* affects the Fermi-surface property, superfluidity, whereas the opposite is not true.

Since QHD is an effective theory of hadrons, form factors or cutoffs related to the spatial size of hadrons might be necessary [22]. On the other hand, an important feature of the gap equation is that it has such a form that a short-range correlation is involved [23,24,9]. This leads to an oscillatory behavior of $\Delta(k)$ similar to that of $\bar{v}_{pp}(k, k_F)$ as functions of k [8,15,16]. Matera *et al.* found that their formulation for the pairing gap gave a cutoff [13]. This is interesting in the respect that both, which originate from the pairing correlation, act to evade repulsion at high momentum. They reported that the values they obtained were $1.7\text{--}1.9 \text{ fm}^{-1}$. These values mean that the repulsive part was completely cut; see Fig. 2. Since our formulation does not have any means to choose a cutoff as those of Refs. [5,12], here we show the dependence of $\Delta(k_F)$ at $k_F = 0.9 \text{ fm}^{-1}$, where it becomes maximum, on the cutoff momentum Λ_c as a free parameter. The result is presented in Fig. 3. This shows that the MFT result decreases steeply as the cutoff decreases. This can be understood as follows: The difference between the RHA and the MFT in the low-density case in Fig. 1 mainly comes from the difference in the magnitude of the high-momentum repulsion. In other words, the contribution of the high-momentum repulsion is more important in the MFT case. Consequently, as the cutoff momentum decreases, the $\Delta(k_F)$ of the MFT calculation decreases more. The plateaus around $\Lambda_c = 2\text{--}3 \text{ fm}^{-1}$ in the RHA and $\Lambda_c = 1\text{--}2 \text{ fm}^{-1}$ in the MFT are due to the sign change of v_{pp} around there (see Fig. 2). A comparison with the Bonn-B potential [25] which reproduced the maximum pairing gap accepted in nonrelativistic studies [15,3] is also shown in Fig. 2. This indicates that the high-momentum repulsion of the MFT without a cutoff is too strong and the low-momentum repulsion is too weak both in the RHA and in the MFT to give reasonable pairing gaps. This is because the parameters of the σ - ω model are adjusted to mean-field calculations to which only momenta below the

Fermi surface contribute as pointed out in Ref. [5]. A possible improvement in the p-p channel would be to take into account the nucleon-antinucleon ($N\bar{N}$) polarization. Our MFT case corresponds to the no-sea σ - ω case in Fig. 4 of Ref. [12], but the gap values there are about twice larger than ours. The origin of this difference might be the Fock contributions.

To summarize, we have studied two kinds of Dirac sea effects on the pairing gap in nuclear matter. One is the vacuum fluctuation effect on the nucleon effective mass and the other is the direct coupling between the Fermi sea pairing

and the Dirac sea pairing. The former is a bulk effect while the latter affects only the Fermi surface. Our calculation indicates that the former is more important. The dependence of $\Delta(k_F)$ on the high-momentum cutoff was also discussed and the MFT result has been shown to depend more strongly on it than the RHA one.

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