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RELATIVISTIC STUDY OF MEDIUM POLARIZATION EFFECTS ON SUPERFLUIDITY IN NUCLEAR MATTER

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Effects of the Fermi-sea polarization on the superfluidity in nuclear matter are studied in a framework of quantum hadrodynamics. The particle-hole polarization due to σ and ω mesons enhances the peak value of the pairing gap contrary to the previous non-relativistic studies.

1 Introduction

Pairing correlation in nuclear matter has long been studied mainly in neutron matter from a view point of neutron-star physics such as cooling rates¹. In addition, pairing in nuclear matter with finite Z/N ratio is also becoming of interest as a basic information for the structure theory of finite nuclei, since recent development of RI-beam experiments makes it possible to study $N \simeq Z$ medium-heavy nuclei and neutron-rich light nuclei.

At present, there are two ways to describe the fundamental properties such as the saturation property of the finite-density nuclear many-body system; the non-relativistic and the relativistic models. They are understood as describing observed properties almost equally. Among them, we here adopt the latter because of its relative proximity to the underlying field theory. The origin of quantum hadrodynamics (QHD) can be traced back to Duerr's relativistic nuclear model² which reformulated a non-relativistic field theoretical model of Johnson and Teller³. Since Chin and Walecka succeeded in reproducing the saturation property of symmetric nuclear matter within the mean-field approximation⁴, QHD has not only been evolving beyond the mean-field approximation as a many-body theory but also been enlarging its objects as a nuclear structure model as infinite matter \rightarrow spherical \rightarrow deformed \rightarrow rotating

nuclei⁵. These successes indicate that the particle-hole interaction in QHD is realistic. In contrast, relativistic nuclear structure calculations with pairing done so far have been using particle-particle interactions borrowed from non-relativistic models and therefore the particle-particle channel in QHD has not been studied well even in infinite matter. Aside from practical successes, this situation is unsatisfactory theoretically. Therefore, in this talk, we present an effort to derive an in-medium particle-particle interaction which is consistent with the relativistic mean field although only infinite matter can be discussed at the present stage.

Up to now, there have been a lot of non-relativistic studies of the pairing in nuclear matter. As for the particle-particle interaction entering into the gap equation, some authors adopted bare interactions whereas others adopted renormalized ones such as G -matrices or with Jastrow correlations. Although, in the medium, renormalized interactions should be used intuitively, following reasons support the use of bare interactions: 1) The Green's function formalism leads to the sum of the irreducible diagrams^{6,7}, and its lowest order is the bare interaction. 2) The gap equation itself implies the short-range correlation^{8,9,17}. In general, medium renormalizations are expected to enhance the gap by reducing the short-range repulsion. Anyway, as a next step, polarization diagrams should be considered. One of the formulations which incorporate both the ladder and the ring diagrams consistently is Babu-Brown's¹⁰. Some works^{11,12} were done based on this. Among them, ref.11 considered momentums around the Fermi surface only, while ref.12 solved the gap equation in the full momentum range. The other¹³ is based on the correlated basis function method. All these works concluded that the inclusion of the polarization reduced the pairing gap.

On the other hand, there are a few relativistic calculations of the pairing gap. The first one was done by Kucharek and Ring¹⁴. They adopted the one-boson-exchange (OBE) interaction with the ordinary QHD parameters (both the linear and the non-linear sets), which gave the saturation, under the no-sea approximation. The resulting maximum pairing gap given by the full-range gap equation was about three times larger than the accepted values in the non-relativistic calculations. It should be noted, however, that whether the full-range calculation with effective forces is adequate or not is not trivial as discussed in ref.14. In this respect, Matera et al. discussed a prescription to introduce a cut-off of the high-momentum region¹⁵. Although Guimarães et al. reported that the inclusion of the negative-energy states improved the result¹⁶, here we concentrate on the effects of the Fermi-sea polarization to differentiate pure relativistic effects from those common to non-relativistic models.

2 Outline of formulation

As described in ref.14, meson fields also have to be treated quantum mechanically to incorporate the pairing field via the anomalous (Gorkov) Green's functions. The resulting Dirac-HFB equation reduces to the ordinary BCS equation in the infinite matter case. Therefore, the actual task is to solve the coupled equations:

$$\begin{aligned} M^* &= M - \frac{g_s^2}{m_s^2} 2\pi^2 \int_0^\Lambda \frac{M^*}{\sqrt{k^2 + M^{*2}}} v^2(k) k^2 dk, \\ \Delta(p) &= -\frac{1}{8\pi^2} \int_0^\Lambda v_{pp}(p, k) \frac{\Delta(k)}{\sqrt{(e_k - e_{kp})^2 + \Delta^2(k)}} k^2 dk, \\ v^2(k) &= \frac{1}{2} \left(1 - \frac{e_k - e_{kp}}{\sqrt{(e_k - e_{kp})^2 + \Delta^2(k)}} \right), \\ e_k &= \sqrt{k^2} + M^{*2} + g_v V_0, \end{aligned} \quad (1)$$

where $v_{pp}(p, k)$ is an angle-integrated, anti-symmetrized matrix element of the adopted particle-particle interaction, Λ is a cut-off momentum, and $\gamma=4$ (2) for symmetric nuclear matter (neutron matter). A natural choice of v_{pp} is the OBE without form factors, with the coupling constants and meson masses which are consistent with the mean field. Note that the mesons whose ground-state expectation values are zero (such as π) can also contribute here. Since it was shown in ref.14 that the pairing field was determined mainly by σ and ω , however, we take into account only these two in the present polarization calculation.

Here we examine the Fermi-sea polarization diagrams added to the OBE terms mentioned above. The polarization term due to σ , for example, is given by

$$v_{pp}^{(pol)}(p, k) = \left(\frac{g_s}{q^2 - m_s^2} \right)^2 \Pi^\sigma(q) (\bar{u}u)_1 (\bar{u}u)_2, \quad (2)$$

with a 4-momentum $q = k - p$, and u stands for the nucleon spinor with tildes indicating time-reversal. Similar terms due to ω and the σ - ω mixing are also included. As for the polarization insertion $\Pi(q)$, we examine the ordinary Feynman-density decomposition and the particle-hole-antiparticle decomposition¹⁷ in order to single out the pure particle-hole effects which compare with those in the non-relativistic calculations. In the actual calculation, an instantaneous approximation ($q^0 = 0$) was adopted as in refs.14 and 15. Note that a spin sum, an angle integration and an anti-symmetrization are necessary to obtain the matrix elements in the 1S_0 gap equation in (1). The following results are given by a one-ring calculation. A preliminary RPA calculation shows that the collectivity does not change the essential feature.

3 Results and discussion

Figure 1 shows the result of the inclusion of the p-h polarization. The solid line indicates the pairing gap in neutron matter as a function of the Fermi momentum given by the OBE interaction with the ordinary QHD-1 parameter set. As mentioned earlier, the peak value is about three times larger than the accepted values in the non-relativistic studies. The dashed line indicates the result given by the OBE + pure p-h polarization. This shows that the inclusion of the p-h polarization enhances the peak value while the density range of the superfluid phase is reduced. Contrary to our expectation, this result is completely opposite to the non-relativistic one shown in fig.5 in ref.12, for example. This is because $v_{pp}^{(pol)}$, which is given by a cancellation between the attractions due to σ and due to ω and the repulsion due to the σ - ω mixing, is attractive at low densities while it is repulsive at high densities as shown in fig.2. Further inclusion of the Pauli blocking, which is realized by adopting the polarization insertion given by the density part of the Feynman-density decomposition, enhances the gap especially at high densities as indicated by the dotted line in fig.1.

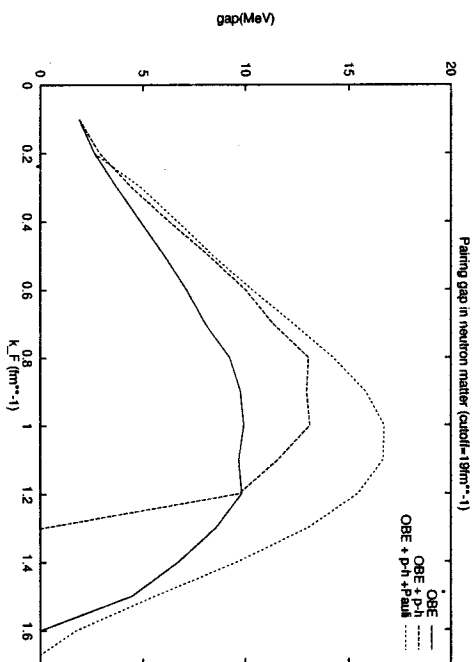


Figure 1: Pairing gap in neutron matter is shown as a function of the Fermi momentum. The solid, the dashed and the dotted lines indicate the OBE, the OBE + p-h polarization and the OBE + p-h polarization + Pauli blocking cases, respectively. The second and the third ones were given by the p-h-a and the Feynman-density decompositions, respectively. Parameters used are $\Lambda = 19 \text{ fm}^{-1}$, $m_\sigma = 520 \text{ MeV}$, $m_\omega = 783 \text{ MeV}$, $g_\sigma = 9.051$ and $g_\omega = 11.672$.

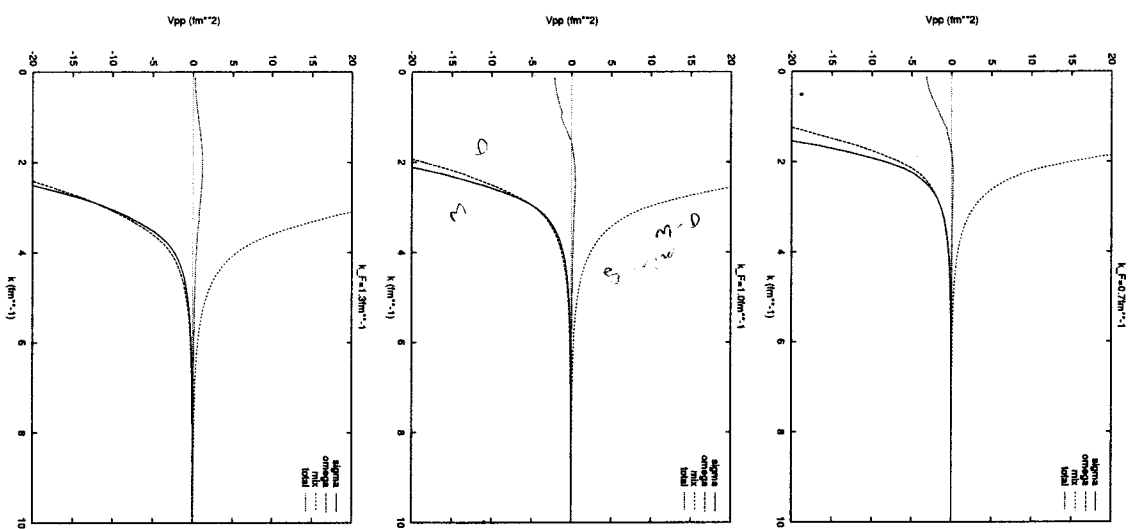


Figure 2: The particle-particle interaction matrix element stemming from the p-h polarization, $v_{pp}^{(pol)}(k, k_F)$, is shown as a function of the momentum k for the Fermi momentum $k_F = 0.7, 1.0$ and 1.3 fm^{-1} from top. 'total' is given by a strong cancellation between the attractions due to σ and due to ω and the repulsion due to the σ - ω mixing.

Before drawing definite conclusions from the present result, two other ingredients should be considered. One is to include other mesons (such as π and ρ) although in the OBE level they were of no importance. This is suggested by an early non-relativistic study of finite nuclei that the polarization due to the tensor force gave a strong repulsion in the 1S_0 channel¹⁸. The other is to include the Dirac-sea polarization. This is suggested by the work of Guimarães et al. which included the negative-energy states in the OBE level¹⁶ and by the study of Friman and Henning that the $N-\bar{N}$ polarization changed the sign of $\Pi(\vec{q})$ ¹⁹, which will produce a sign change of $v_{pp}^{(pol)}$ in the present case. Among these two ingredients which may influence the present result, if the former is dominant the positive-energy sector of relativistic models contains similar physics to non-relativistic models, whereas if the latter is dominant the positive-energy sector alone cannot compare with non-relativistic models.

Finally, since QHD is an effective theory for hadronic many-body systems, some high-momentum cut-off related to the nucleon size may be necessary. Numerical results of the coupled eqs.(1) converge around $\Lambda \simeq 10 \text{ fm}^{-1}$. The result presented in fig.1 was of $\Lambda = 19 \text{ fm}^{-1}$, this can be regarded as a full-range calculation. We examined also some smaller values of Λ . As discussed in ref.14, it is possible to reduce the absolute magnitude of the pairing gap by cutting off the high-momentum contributions. A prescription to determine Λ was proposed in ref.15. Since the p-h polarization is a low-momentum process, the qualitative feature that it enhances the peak value of the gap is not changed.

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