# CONFERENCE ON PHYSICS FROM LARGE $\gamma$-RAY DETECTOR ARRAYS

## Clark Kerr Campus
## Berkeley, California

## August 2-6, 1994

## Volume II
## Proceedings




LBL35687
CONF-940888



Nuclear Science Division, Lawrence Berkeley Laboratory
University of California, Berkeley, CA 94720



This work was supported by the Director, Office of Energy Research, Office of Basic Energy Sciences, of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

# Microscopic Study of Wobbling Motions

Yoshifumi R. Shimizu and Masayuki Matsuzaki*)
*Department of Physics, Kyushu University, Fukuoka 812, Japan*
*) *Department of Physics, Fukuoka University of Education, Munakata, Fukuoka 811-41, Japan*

Abstract
The nuclear wobbling motion is investigated from a microscopic viewpoint. It is shown that the expressions of not only the excitation energy but also the E2 transition rate in the microscopic RPA framework can be cast into the corresponding forms of the macroscopic rotor model. The condition that the microscopic RPA solution can be interpreted to be the wobbling motion is clarified.

## §1 Introduction

In this talk We would like to discuss the wobbling motion[1], which is a collective motion expected in nuclei by an analogy with the classical motion of the asymmetric top. It is a kind of the three dimensional motion in the sense that the rotation axis and the inertia axis of the body do not coincide. Quite recently the "tilted axis cranking" scheme has been proposed[3]. It might be instructive to compare characteristic features of these two. They are similar since the rotation axis deviates from the inertia axis, but these two are conceptually different as in the following, although it might be difficult to distinguish in the experimental data. In the wobbling motion the angular momentum vector and the angular frequency vector are not parallel in the body-fixed frame of coordinate and then the motion is not stationary. Hence the two vectors draw some trajectories even in the body-fixed frame. In contrast the angular momentum and the angular frequency vectors are parallel and as a result the rotation is stationary and uniform in the tilted axis cranking. When quantized the tilted axis cranking gives a description for an isolated band just like the usual cranking does, but the wobbling motion generally corresponds to a multiple band structure in the quantal spectra.

There is a basic vacuum band, e.g. the yrast band, from which excited bands are generated just like the multi-phonon structure; one wobbling phonon excited band, two wobbling phonon excited band, and so on. In each bands (horizontal sequences) states are connected by the strong rotational $E2$ transitions, while these phonon bands are also vertically connected by $\Delta I = \pm 1$ $E2$ or $M1$ transitions in general. Here the vacuum band is just described by the usual cranking, i.e. the uniformly rotating states around the inertia axis of the largest moment of inertia, but when the wobbling phonons are excited the rotation axis deviates from the inertia axis more and more. Since higher excited bands are difficult to access in experiments, we concentrated on the first wobbling band and discuss the characteristic feature of the $\Delta I = \pm 1$ $E2$ transition to the vacuum band in the following.

## §2 Microscopic formulation

Now how to calculate such a excited band like the wobbling phonon band? We use the microscopic formalism[2] based on the random phase approximation (RPA), which is generally known to be suitable for describing the vibrational motions. For the first excited wobbling band, the deviation of the angular momentum vector from the usual



cranking axis, $x$-axis, is expected to be small and the excitation mode transfer the angular momentum by $\pm 1$ unit so that it has definite signature $r = -1$, or $\alpha = 1$.

The excitation energy is given by the well-known RPA eigen-value equation. Since the equation generally gives many solutions, most of which are of non-collective nature, we denote the n-th eigen-value, $\hbar\omega_n$:

$$[H - \hbar\omega_{\rm rot} J_x, X_n^\dagger]_{\rm RPA} = \hbar\omega_n X_n^\dagger, \tag{1}$$

where $X_n^\dagger$ is the n-th phonon creation operator and as for the microscopic hamiltonian we use the cranked-Nilsson single-particle potential and the pairing plus $QQ$ residual interactions. Another important observable is the $\Delta I = \pm 1$ $E2$ transitions from the n-th RPA phonon excited state to the vacuum state, the transition energy of which is is given by $E_\gamma(n) = \hbar\omega_n \mp \hbar\omega_{\rm rot}$, and can be quite simply calculated in the high-spin limit:

$$B(E2)^{\rm inter}_{\Delta I=\pm 1} \approx \frac{1}{2}|\langle f|(Q^{(-)}_{K=1} \pm Q^{(-)}_{K=2})^{(E)}|i\rangle|^2 \to \frac{1}{2}\left((\mathcal{Q}_y(n) \mp \mathcal{Q}_z(n))^{(E)}\right)^2, \tag{2}$$

where the transition operator is composed of the signature-coupled quadrupole operators with $K$ (with respect to $z$-axis) = 1 and 2 and $r = -1$, and explicitly written as

$$Q^{(-)}_{K=1} = -\frac{1}{2}\sqrt{\frac{15}{4\pi}}\sum_{a=1}^{A}(xz)_a \equiv Q_y, \quad Q^{(-)}_{K=2} = i\frac{1}{2}\sqrt{\frac{15}{4\pi}}\sum_{a=1}^{A}(xy)_a \equiv Q_z. \tag{3}$$

The transition matrix elements are evaluated by a commutator with the phonon annihilation operator, $\mathcal{Q}_{y,z}(n) \equiv [X_n, Q_{y,z}]_{\rm RPA}$, and are denoted by script Q's. Here we introduced the notation, $Q_y$ and $Q_z$ for these non-diagonal part of the quadrupole tensor. This is because these operators describe the shape fluctuations around $y$, $z$-axes. Actually, by taking the commutation relations with the corresponding angular momentum operator, $J_{y,z}$,

$$[Q_y, iJ_y]_{\rm RPA} = \sqrt{\frac{15}{4\pi}}\sum_{a=1}^{A}\langle(x^2 - z^2)_a\rangle \equiv 2R^2\alpha_y, \tag{4}$$

$$[J_z, Q_z]_{\rm RPA} = \sqrt{\frac{15}{4\pi}}\sum_{a=1}^{A}\langle(x^2 - y^2)_a\rangle \equiv 2R^2\alpha_z, \tag{5}$$

we get the diagonal part of the quadrupole tensor, or the static deformations around $y, z$-axes. Here we denote the corresponding parameters, $\alpha_y \approx \sqrt{\frac{5}{9\pi}}\epsilon_2 \sin(\gamma + \frac{4\pi}{3})$ and $\alpha_z \approx -\sqrt{\frac{5}{9\pi}}\epsilon_2 \sin\gamma$. As is well-known, the in-band $\Delta I = \pm$ $E2$ transitions can be calculated by these static deformations for both the vacuum and the wobbling bands within the RPA as

$$B(E2)^{\rm in-band}_{\Delta I=\pm 2} \approx \left(e\frac{Z}{A}\right)^2\frac{1}{2}R^4(\alpha_y - \alpha_z)^2. \tag{6}$$

Note that in the ground state, where no rotation is imposed, either $\mathcal{Q}_y$ or $\mathcal{Q}_z$ amplitudes is zero. If the system is cranked $\Delta K = \pm 1$ $K$-mixing induced by the rotation makes both amplitude non-zero at the same time. Then the transition rates with $\Delta I = +1$ and $-1$ transitions can be different and show staggering just like the well-known staggering of $B(M1)$ between the signature partner bands in odd nuclei[5].

From the calculational point of view the discussion until now is enough. But then how the wobbling picture comes out? One must make a (time-dependent) coordinate transformation in order to see this. Though not mentioned explicitly, we have worked out, up to now, in the so-called uniformly rotating (UR) frame. There the rotation is still around the $x$-axis. It is, however, not the appropriate coordinate system because the (time-dependent) shape-fluctuations are induced by the excitation of the wobbling phonon so that the quadrupole tensor of the system is non-diagonal. As in the description of the rigid-body, we naturally come to the so-called principal axis (PA) frame[2] by diagonalizing the quadrupole tensor. Then the shape fluctuations disappear and, in place of it, the coordinate transformation makes the angular momentum and angular frequency vectors to wobble around the $x$-axis. Namely, in terms of the time-dependent mean-field theory, the single particle hamiltonian corresponding to the state with the wobbling phonons excited is

$$h_{\rm UR}(t) = h_{\rm def} - \hbar\omega_{\rm rot} J_x - \kappa_y \mathcal{Q}_y(t) Q_y - \kappa_z \mathcal{Q}_z(t) Q_z, \qquad (7)$$

in the UR frame, while

$$h_{\rm PA}(t) = h_{\rm def} - \hbar\omega_{\rm rot} J_x - \hbar\omega_y(t) J_y - \hbar\omega_z(t) J_z, \qquad (8)$$

in the PA frame, where $\omega_{\rm rot} \approx \omega_x$ in the small amplitude RPA approximation. Since the angular momentum vector is not parallel to the angular frequency vector, three moments of inertia can be defined in the PA frame, as is usual, by $\mathcal{J}^{\rm eff}_{x,y,z}(n) = (I_{x,y,z}(n))_{\rm PA}/\hbar\omega_{x,y,z}(n)$ where the $(n)$ denotes that $\boldsymbol{I}$ and $\boldsymbol{\omega}$ vectors are evaluated with respect to the n-th RPA solution. Note $\mathcal{J}^{\rm eff}_x \approx \mathcal{J}_x \equiv I/\hbar\omega_{\rm rot}$, where $I$ is the spin of the vacuum cranked state, again within the small amplitude approximation.

Using the three moments of inertia thus introduced, it can be shown that the excitation energy is written as[2],

$$\hbar\omega_n = I\sqrt{W_y(n)W_z(n)} \quad \text{with} \quad W_{y,z}(n) \equiv 1/\mathcal{J}^{\rm eff}_{y,z}(n) - 1/\mathcal{J}_x, \qquad (9)$$

and the $E2$ transition is as[6],

$$B(E2)^{inter}_{\Delta I=\pm 1} \approx \left(e\frac{Z}{A}\right)^2 \frac{1}{I} R^4 c_n^2 \left(\alpha_y \left(\frac{W_z(n)}{W_y(n)}\right)^{1/4} \mp \sigma_n \alpha_z \left(\frac{W_y(n)}{W_z(n)}\right)^{1/4}\right)^2, \qquad (10)$$

where $(\mathcal{Q}_{y,z})^{(E)} = (eZ/A)\mathcal{Q}_{y,z}$ is assumed. Notice that these expressions formally coincide with those given by the macroscopic rotor model, except the overall factor $c_n^2$ and the sign $\sigma_n$ for the $E2$ transitions probability, namely, if $c_n^2 = 1$ and $\sigma_n = +$ then it exactly coincides. Let us call the microscopic RPA solution wobbling-like if it (approximately) satisfies these conditions. Generally, $c_n^2$ is not exactly 1 microscopically even for the ideal case* because there are many RPA solutions in

*Actually, a kind of "some rule", $\sum_{n \neq NG} c_n^2 \sigma_n = 1$, can be proved[6].



contrast to the simple rotor model, but the presence of the definite sign factor $\sigma_n = +$ gives us an important phase rule. Comparing with the original expression eq.(2), relative sign between the static and the dynamic deformation coincide if the sign factor $\sigma_n = +$;

$$\text{sign of } (\mathcal{Q}_y(n)/\mathcal{Q}_z(n)) = \text{sign of } (\alpha_y/\alpha_z). \qquad (11)$$

Thus, the zigzag behaviour of the $\Delta I = \pm 1$ $E2$ transitions, in addition to the energy spectra, reflect the behaviours of both the triaxiality of the mean-field and the three moments of inertia. Especially, as a result, for the wobbling-like RPA solutions, which transition probability is larger, i.e. one with $I \to I+1$ or $I \to I-1$, is determined solely by the triaxiality. The relations are schematically summarized in Fig.1.

It should be mentioned that the appreciable amount of deformation around $y$, $z$-axes are necessary, $\alpha_y \neq 0$ and $\alpha_z \neq 0$, in order for the transformation to be perform from the UR to the PA frame: Namely, the small amplitude ansatz of the "wobbling" motion in the PA frame should be satisfied[6],

$$\begin{aligned} O(1/\sqrt{I}) &\sim i(I_y(n))_{\rm PA}/I = \mathcal{Q}_z(n)/2R^2\alpha_z, \\ O(1/\sqrt{I}) &\sim -(I_z(n))_{\rm PA}/I = \mathcal{Q}_y(n)/2R^2\alpha_y. \end{aligned} \qquad (12)$$

This equation also shows that the amplitude of the "wobbling" of the angular momentum vector in the PA frame is related to the fluctuations of deformation around $y, z$-axes, i.e. the ratios of the dynamic and the static deformations, in the UR frame.

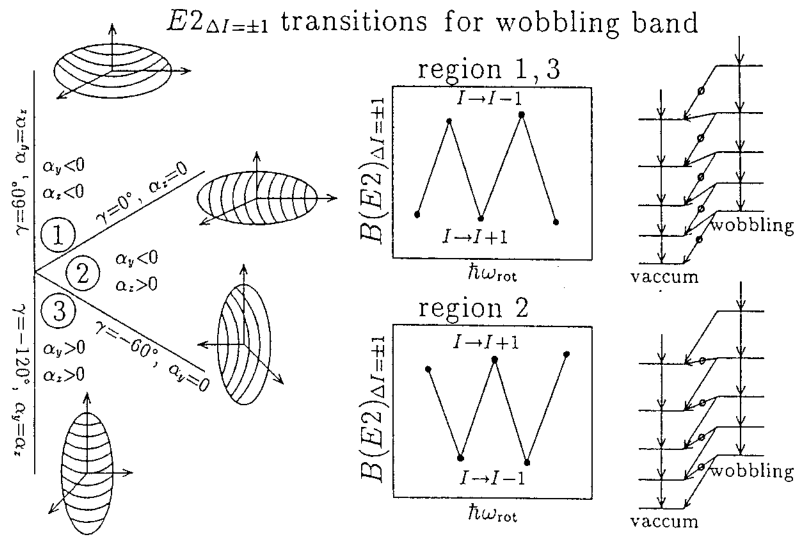


Figure 1: Schematic figure depicting the relation between the triaxiality of the mean-field and the $\Delta I = \pm$ $E2$ transitions from the wobbling band to the vacuum band. The transitions with stronger $B(E2)$'s are marked in the spectra (right pannels).

## §3 Discussions

In contrast to the case of the $M1$ transitions between the signature partner bands in odd nuclei, the zigzag behaviours of the $\Delta I = \pm 1$ interband $E2$ transitions in even-even nuclei are not observed so often. The only exceptions are the transitions between the well-known $\gamma$ band and the ground-state band at low-spin. It has been shown[6] that our basic formulation can be applicable also in such cases and gives satisfactory agreements with data.

Encouraged with these results, we have performed some realistic calculations[6] at higher-spins, where the $\Delta K = \pm 1$ mixing effects caused by the Coriolis interaction and the alignments of quasiparticles are expected to more favour the appearance of the wobbling-like collective motions. One of candidates of the wobbling band has been known[4,5] in $^{182}$Os. We have found another possible candidate in $^{124}$Xe, whose yrast is the s-band (two neutron quasiparticles aligned) after $I > 8\hbar$ and expected to have $\gamma \approx -45°$. As in the case of $^{182}$Os this nucleus belongs to the region 2 in Fig.1 and, therefore, has stronger $I \rightarrow I+1$ $B(E2)$'s. It should be mentioned, however, the $M1$ transitions are non-negligible in this nuclei in contrast to the case of $^{182}$Os, see ref.6) for details.

Interestingly enough, as far as we have studied, when the collective RPA solution exists the lowest always satisfies the condition of the wobbling-like solution, eq.(11). However, it has been shown[6] at the same time that the microscopically calculated $\gamma$-dependence of the three moments of inertia is neither irrotational nor rigid-body like. Therefore the microscopic properties of the nuclear wobbling motion is not so simple as is expected from the macroscopic rotor model. As an example, the $M1$ transitions can be very strong depending on the quasiparticle configuration of the vacuum band, e.g. whether quasineutrons or quasiprotons are aligned. It should be noticed that important information of the triaxiality and the three moments of inertia can be extracted from the combined use of both the energy spectra, eq.(9), and the ratio between the in-band, eq.(6), and the interband, eq.(10), $B(E2)$'s.

The $\Delta I = \pm 1$ $E2$ transitions under discussions are typically ten times the Weisskopf unit. Although largely enhanced, they are still an order of magnitude smaller compared to the in-band rotational $E2$ transitions. We hope that new generations of the large array of the crystal ball will give us more detailed information of the electromagnetic transitions, which are necessary to confirm our predictions and to identify the nuclear wobbling motion if they exist.

## References


1) Å. Bohr and B. R. Mottelson, *Nuclear Structure*, Vol. II, (Benjamin, New York, 1975), Chap. 4, p.158ff.

2) E. R. Marshalek, Nucl. Phys. **A331** (1979), 429.

3) S. Frauendorf, Nucl. Phys. **A557** (1993), 257c; the talk in this conference.

4) M. Matsuzaki, Nucl. Phys. **A509** (1990), 269.

5) Y. R. Shimizu and M. Matsuzaki, in the Proceedings of the International Conference on *Nuclear Structure at High Angular Momentum*, May 18-21, 1992, Ottawa, AECL-10613, pp.278-282.

6) Y. R. Shimizu and M. Matsuzaki, to be published.