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Three-dimensional collective rotation and intrinsic motion in relativistic many-body systems

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Three-dimensional collective rotation and intrinsic motion in relativistic many-body systems are described by a relativistic quantum field theory with the Lagrangian containing nucleons and mesons. In this reasoning, the cranking model in uniform rotation is extended to the case of non-uniform rotation with a time-dependent cranking term. Since a rotating frame is an accelerated one, the technique of general relativity is used. The body-fixed frame is determined by imposing constraints, which correspond to the gauge-fixing conditions in the gauge theory. A canonical formulation of three-dimensional rotation and intrinsic motion is derived from this constrained system. The quantization of the classical system is performed using the Dirac procedure.

In recent years, experimental data on the high-spin domain of deformed nuclei above the yrast line has increased. In a non-relativistic formulation, the self-consistent cranking (SCC) model [1] is useful in describing microscopically the yrast states of uniformly rotating nuclei. For non-uniform rotation, several authors have proposed the use of the random phase approximation (RPA) method [2,3], which describes small amplitude fluctuations of the yrast states. In the RPA, the deviations from uniform rotation are described as small oscillations of the rotational axis, or so-called wobbling motions. However, this method is limited to small amplitude collective motion. It is simpler to describe non-uniform rotation from a moving frame by introducing the Euler angles. This method is considered to be a generalization of the cranking model in the case of non-uniform rotation. However, the degrees of freedom of the system are overcomplete due to the introduction of the Euler angles, and consequently zero modes arise which leads to infrared divergences [2,4]. When we go beyond the RPA to higher orders, the conventional perturbation method fails due to the presence of these zero modes. Therefore, constraints to determine the intrinsic frame microscopically are needed in order to eliminate these zero modes. One of the present authors (K.K.) has recently proposed a method [5], based on the time-dependent Hartree-Fock (TDHF) theory, which differs from the approach of Bes, Kurchan, and Barrios [6], and Kerman and Onishi [7], to describe collective rotation and intrinsic motion beyond the RPA to higher orders. Then, the motions on the TDHF submanifold were completely separated into three-dimensional collective rotation and intrinsic motion by imposing the constraints.

A relativistic quantum field theory, which consists of nucleons and mesons, has recently been developed to study nuclear many-body problems [8,9]. The mean-field theory (MFT), which was proposed by Walecka [8], can reproduce the bulk properties of doubly magic nuclei well. One of the present authors (M.N.) and Hasegawa [10] recently have presented a fully quantum-mechanical treatment of a spherical finite nuclear system on the basis of the Schwinger–Dyson formalism. The MFT was applied to axial-symmetric deformed nuclei [11], and it was shown that it can be used to describe collective rotation. Furthermore, Koepf and Ring [12] have studied uniform rotation in the framework of the MFT. Their model described the axial-symmetric deformed nucleus based on the cranking model with constant angular velocity. They investigated the yrast line of the deformed nucleus ^{20}Ne, and obtained a good agreement with experimental data: the binding energy, RMS radii, and quadrupole moments. Furthermore, they investigated superdeformed shapes in rapidly rotating nuclei ^{80}Sr and ^{152}Dy [13]. In their calculation, they found the ground state of ^{80}Sr to be triaxial deformation, and a smooth increase of the deformation parameter γ with total spin. However, their method is restricted one-dimensional cranking in spite of triaxial deformation. The cranking method is basically classical and can be understood as an approximation to a fully quantum-mechanical description. Then, we can expect quantum fluctuations around the three-dimensional cranking solution.

In this paper we will apply our method in the non-relativistic formulation to the relativistic many-body system in non-uniform rotation, and present a complete and consistent theoretical treatment of the quantum fluctuation and the intrinsic motion. Then, it will be shown that the separation of the three-dimensional rotation and the intrinsic motion is also done in the relativistic many-body system.

We first start from the Landau-gauge Lagrangian density in the laboratory frame:

$$\mathscr{L}=\bar{\psi}(\gamma^{\mu}\mathrm{i}\partial_{\mu}-m-g_{\sigma}\sigma-g_{\omega}\gamma^{\mu}V_{\mu})\psi+\tfrac{1}{2}(\partial_{\mu}\sigma\partial^{\mu}\sigma-m_{\sigma}^{2}\sigma^{2})-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}m_{\omega}^{2}V_{\mu}V^{\mu}+B\partial_{\mu}V^{\mu}\,, \tag{1a}$$

$$F_{\mu\nu}=\partial_{\mu}V_{\nu}-\partial_{\nu}V_{\mu}\,, \tag{1b}$$

where ψ, σ and V_{μ} represent the nucleons with mass m, σ-meson with mass m_{σ} and ω-meson with mass m_{ω}, respectively. The ψ, σ and V_{μ} are described by the spinor (Grassmann) field, the scalar field and the vector field, respectively, and B is the auxiliary scalar field due to the Landau gauge. The nucleons and the mesons interact with each other through linear meson couplings. The nucleon mass m and the ω-meson mass m_{ω} are usually given by experimental values. The coupling constants g_{σ}, g_{ω} are determined by fitting to both the nuclear matter characteristics and some of the ground state properties of nuclei. Since the Lagrangian of a massive vector meson field does not have a local U(1) gauge invariance, in general it is not necessary to introduce the auxiliary field B. However, if the canonical procedure is carried out, there is a shortcoming, because V_{k} and V_{0} do not commute at the same time. This gives rise to some serious problems, which are obviated by introducing the auxiliary field B.

From the variational principle for the Lagrangian density, the equations of motion are given as

$$[-\mathrm{i}\alpha\cdot\nabla+g_{\omega}V_{0}+\beta(m+g_{\sigma}\sigma)]\psi=\mathrm{i}\partial_{0}\psi\,,\quad \Box\sigma+m_{\sigma}^{2}\sigma+g_{\sigma}\rho=0\,,\quad (\Box+m_{\omega}^{2})V^{\mu}-\partial^{\mu}B=g_{\omega}j_{V}^{\mu}\,, \tag{2a,b,c}$$

$$\partial_{\mu}V^{\mu}=0\,. \tag{2d}$$

$$\Box\equiv\partial_{\mu}\partial^{\mu}=\partial_{0}^{2}-\Delta\,,\quad \rho=\bar{\psi}\psi\,,\quad j_{V}^{\mu}=\bar{\psi}\gamma^{\mu}\psi\,, \tag{2e}$$

where $\Box$ and Δ are the d'Alembertian and Laplacian, respectively. Eq. (2a) is the Dirac equation for nucleons, and (2b), (2c) are the Klein–Gordon equations for mesons. Eq. (2d) is the Lorentz condition.

The canonical formulation is obtained from the canonical conjugate fields defined as

$$\Pi_{i}\equiv\frac{\partial\mathscr{L}}{\partial\dot{q}_{i}}\,,\quad q_{i}\equiv(\psi,\sigma,V)\,, \tag{3a}$$

and then the Π_{i} are given by

$$\Pi=\mathrm{i}\psi^{\dagger}\,,\quad \Pi_{\sigma}=\dot{\sigma}\,,\quad \Pi_{k}=F_{0k}\quad (k=1,2,3)\,,\quad \Pi_{0}=B\,, \tag{3b}$$

where the dot denotes the time derivative. As is well known, there are two kinds of derivative due to the anti-commutation of the Grassmann number: right derivative and left derivative. Hereafter, we will use the right derivative. Thus, we can obtain the Hamiltonian density

$$\mathscr{H}=-\mathrm{i}\psi^\dagger\alpha\cdot\nabla\psi+\psi^\dagger\beta(m+g_\sigma\sigma)+g_\omega\psi^\dagger(V^0-\alpha^kV^k)\psi+\tfrac{1}{2}(\Pi_\sigma^2+\partial_i\sigma\,\partial_i\sigma+m_\sigma^2\sigma^2)+\tfrac{1}{2}\Pi_i^2+\tfrac{1}{4}F_{ij}^2$$
$$+\tfrac{1}{2}m_\omega^2V_\mu^2-(\partial_kB)V_k+\partial_k(F_{0k}V_0+BV_k)\,, \tag{3c}$$

where the Greek subscript denotes the four components μ, $\nu=0, ..., 3$, and the Latin one denotes the spatial component $i, j, k=1, ..., 3$. Then, the Hamiltonian is given by

$$H=\int \mathrm{d}^3x[-\mathrm{i}\psi^\dagger\alpha\cdot\nabla\psi+\psi^\dagger\beta(m+g_\sigma\sigma)+g_\omega\psi^\dagger(V^0-\alpha^kV^k)\psi+\tfrac{1}{2}(\Pi_\sigma^2+\partial_i\sigma\,\partial_i\sigma+m_\sigma^2\sigma^2)+\tfrac{1}{2}\Pi_i^2+\tfrac{1}{4}F_{ij}^2$$
$$+\tfrac{1}{2}m_\omega^2V_\mu^2-(\partial_kB)V_k]\,, \tag{4}$$

where the last term of eq. (3c) does not contribute in the above integration. The canonical quantization is carried out by setting the following commutation relations with equal time:

$$\{\psi(x_0,\boldsymbol{x}),\Pi(x_0,\boldsymbol{y})\}=\mathrm{i}\delta(\boldsymbol{x}-\boldsymbol{y})\,,\quad \{\psi(x_0,\boldsymbol{x}),\psi(x_0,\boldsymbol{y})\}=\{\Pi(x_0,\boldsymbol{x}),\Pi(x_0,\boldsymbol{y})\}=0\,, \tag{5a,b}$$

$$[\sigma(x_0,\boldsymbol{x}),\Pi_\sigma(x_0,\boldsymbol{y})]=\mathrm{i}\delta(\boldsymbol{x}-\boldsymbol{y})\,,\quad [\sigma(x_0,\boldsymbol{x}),\sigma(x_0,\boldsymbol{y})]=[\Pi_\sigma(x_0,\boldsymbol{x}),\Pi_\sigma(x_0,\boldsymbol{y})]=0\,, \tag{5c,d}$$

$$[V_\mu(x_0,\boldsymbol{x}),\Pi_\nu(x_0,\boldsymbol{y})]=\mathrm{i}\delta_{\mu\nu}\delta(\boldsymbol{x}-\boldsymbol{y})\,,\quad [V_\mu(x_0,\boldsymbol{x}),V_\nu(x_0,\boldsymbol{y})]=[\Pi_\mu(x_0,\boldsymbol{x}),\Pi_\nu(x_0,\boldsymbol{y})]=0\,. \tag{5e,f}$$

Let us next consider a many-body system of a triaxial deformation. It is simpler to describe it from a three-dimensional rotating frame of reference. The coordinates $x'^\mu=(t', x', y', z')$ in the rotating frame are expressed by the following transformation [14] of the coordinates $x^\mu=(t, x, y, z)$ in the laboratory frame:

$$x'^\mu=M^{\mu\nu}x^\nu\,, \tag{6a}$$

$$M=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & \cos\theta_1\cos\theta_2\cos\theta_3-\sin\theta_1\sin\theta_3 & \sin\theta_1\cos\theta_2\cos\theta_3+\cos\theta_1\sin\theta_3 & -\cos\theta_3\sin\theta_2\\ 0 & -\cos\theta_1\cos\theta_2\sin\theta_3-\sin\theta_1\cos\theta_3 & -\sin\theta_1\cos\theta_2\sin\theta_3+\cos\theta_1\cos\theta_3 & \sin\theta_3\sin\theta_2\\ 0 & \cos\theta_1\sin\theta_2 & \sin\theta_1\sin\theta_2 & \cos\theta_2\end{pmatrix}, \tag{6b}$$

where θ_i are the Euler angles. The Euler angles are the dynamical variables depending on time, and will be determined by intrinsic frame conditions later on.

Since the rotating frame is an accelerated one, we must use the technique of general relativity rather than that of the special relativity. Thus, the covariant metric tensor $g^{\mu\nu}$ is expressed as

$$g^{\mu\nu}=\frac{\partial x'^\mu}{\partial x^\alpha}\eta^{\alpha\beta}\frac{\partial x'^\nu}{\partial x^\beta}=(T\eta T^{\mathrm{T}})^{\mu\nu}\,,\qquad \eta=\begin{pmatrix}1 & \mathbf{0}\\ \mathbf{0} & -\mathbf{1}\end{pmatrix},\quad T=\begin{pmatrix}1 & \mathbf{0}\\ -(\boldsymbol{\Omega}\times\boldsymbol{r}') & \mathbf{1}\end{pmatrix}, \tag{7a,7b}$$

where $\boldsymbol{\Omega}=(\Omega_1, \Omega_2, \Omega_3)$ is the angular velocity vector with respect to the rotating frame and $\boldsymbol{r}'=(x', y', z')$ is the coordinate vector in the rotating frame. The angular velocities Ω_k are expressed by the Euler angles θ_i as follows:

$$\Omega_k=V_{ki}\dot{\theta}_i\,, \tag{8a}$$

where the transformation matrix V is given as

$$V=\begin{pmatrix}-\sin\theta_2\cos\theta_3 & \sin\theta_3 & 0\\ \sin\theta_2\sin\theta_3 & \cos\theta_3 & 0\\ \cos\theta_2 & 0 & 1\end{pmatrix}. \tag{8b}$$

The $\sigma(B)$-field, the V-field, and ψ-field transform like the scalar, the vector, and the spinor, respectively, and then the $\sigma'(B')$-field, the V'-field, and the ψ'-field in the rotating frame are expressed as

$$\sigma'(x')=\sigma(x)\,,\quad B'(x')=B(x)\,,\qquad V'^{\mu}(x')=\frac{\partial x'^{\mu}}{\partial x^{\nu}}V^{\nu}(x)\,, \tag{9a,b}$$

$$\psi'(x')=\Lambda\psi(x)\,,\quad \Lambda=\exp(\mathrm{i}\theta_3 s_z)\exp(\mathrm{i}\theta_2 s_y)\exp(\mathrm{i}\theta_1 s_z)\,, \tag{9c}$$

$$s_x=\tfrac{1}{2}\mathrm{i}\gamma^2\gamma^3\,,\quad s_y=\tfrac{1}{2}\mathrm{i}\gamma^3\gamma^1\,,\quad s_z=\tfrac{1}{2}\mathrm{i}\gamma^1\gamma^2\,, \tag{9d}$$

where s_x, s_y, and s_z are the matrices satisfying the SU(2) algebra

$$[s_i, s_j]=\mathrm{i}\epsilon_{ijk}s_k\,.$$

Since the vector coupling term $\bar\psi\gamma^{\mu}\psi V_{\mu}$ in the Lagrangian (1a) is the Lorentz scalar, we should require that $\bar\psi\gamma^{\mu}\psi$ transforms to a contravariant vector:

$$\bar\psi(x)\gamma^{\mu}\psi(x)=\bar\psi'(x')\Lambda\gamma^{\mu}\Lambda^{\dagger}\psi'(x')\to\frac{\partial x^{\mu}}{\partial x'^{\nu}}\bar\psi'(x')\gamma'^{\nu}(x')\psi'(x')\,. \tag{10a}$$

Therefore, we obtain the following relationship for the gamma matrices depending on the coordinate x'^{μ} in the rotating frame:

$$\gamma'^{\mu}(x')=\frac{\partial x'^{\mu}}{\partial x^{\nu}}\Lambda\gamma^{\nu}\Lambda^{\dagger}=T^{\mu\nu}\gamma^{\nu}\,. \tag{10b}$$

We define the covariant derivative as follows:

$$\mathrm{D}'_{\mu}=\partial'_{\mu}+\Gamma'_{\mu}\,. \tag{11}$$

Since the derivative term $\bar\psi(x)\gamma^{\nu}\partial_{\nu}\psi(x)$ is a scalar, the term should satisfy the relationship

$$\bar\psi(x)\gamma^{\mu}\partial_{\mu}\psi(x)=\bar\psi'(x')\Lambda\gamma^{\mu}\partial_{\mu}\Lambda^{\dagger}\psi'(x')\to\bar\psi'(x')\gamma'^{\nu}\mathrm{D}'_{\nu}\psi'(x')\,. \tag{12}$$

Therefore, we get

$$\Gamma'_{\mu}=\Lambda\partial'_{\mu}\Lambda^{\dagger}-\partial'_{\mu}=\begin{pmatrix}-\mathrm{i}\Omega_k s_k\\0\\0\\0\end{pmatrix}, \tag{13}$$

when we use the relationships

$$\frac{\partial\Lambda^{\dagger}}{\partial\theta_i}=-\mathrm{i}\Lambda^{\dagger}V_{ki}s_k\,. \tag{14}$$

Thus, we can obtain the Lagrangian density in the rotating frame:

$$\mathscr{L}'=\bar\psi'(\gamma'^{\mu}\mathrm{i}\mathrm{D}'_{\mu}-m)\psi'-g_{\sigma}\sigma'\rho'-g_{\omega}V'_{\mu}j'^{\mu}_{V}+\tfrac{1}{2}(g^{\mu\nu}\partial'_{\mu}\sigma'\,\partial'_{\nu}\sigma'-m_{\sigma}^2\sigma'^2)-\tfrac{1}{4}g^{\mu\alpha}g^{\nu\beta}F'_{\mu\nu}F'_{\alpha\beta}$$
$$+\tfrac{1}{2}m_{\omega}^2 g^{\mu\nu}V'_{\mu}V'_{\nu}+B'g^{\mu\nu}\partial'_{\mu}V'_{\nu}\,, \tag{15a}$$

$$\rho'=\bar\psi'\psi',\quad j'^{\mu}_{V}=\bar\psi'\gamma'^{\mu}\psi'\,,\quad F'_{\mu\nu}=\partial'_{\mu}V'_{\nu}-\partial'_{\nu}V'_{\mu}\,. \tag{15b}$$

The variational principle $\delta\int\mathscr{L}'\,\mathrm{d}^4x'=0$ leads to the equations of motion

$$[-\mathrm{i}\alpha\cdot\nabla'+g_{\omega}(\tilde V_0-\alpha^k\tilde V^k)+\beta(m+g_{\sigma}\sigma')-\Omega_k(L_k+s_k)]\psi'=\mathrm{i}\partial_0\psi'\,, \tag{16a}$$

$$(\partial_0-\mathrm{i}\boldsymbol{\Omega}\cdot\boldsymbol{L})(\partial_0-\mathrm{i}\boldsymbol{\Omega}\cdot\boldsymbol{L})\sigma'-\Delta'\sigma'+m_{\sigma}^2\sigma'+g_{\sigma}\rho'=0\,, \tag{16b}$$

$$(\partial_0-\mathrm{i}\boldsymbol{\Omega}\cdot\boldsymbol{L})(\partial_0-\mathrm{i}\boldsymbol{\Omega}\cdot\boldsymbol{L})\tilde V^0-\Delta'\tilde V^0+m_{\omega}^2\tilde V^0-\partial_0 B'=g_{\omega}\tilde j^0_V\,, \tag{16c}$$

$$[\delta_{ik}(\partial_0 - i\boldsymbol{\Omega}\cdot\boldsymbol{L}) - i(\boldsymbol{\Omega}\cdot\boldsymbol{S})_{ik}][\delta_{kl'}(\partial_0 - i\boldsymbol{\Omega}\cdot\boldsymbol{L}) - i(\boldsymbol{\Omega}\cdot\boldsymbol{S})_{kl'}]\tilde{V}^{l'} - \Delta'\tilde{V}^i + m_\omega^2\tilde{V}^i - \partial'^i B' = g_\omega \tilde{j}^i_V, \tag{16d}$$

$$\partial_0\tilde{V}^0 - i\boldsymbol{\Omega}\cdot\boldsymbol{L}\tilde{V}^0 + \partial'_k\tilde{V}^k = 0, \tag{16e}$$

$$\tilde{V}^\mu = (T^{-1}V')^\mu, \quad \tilde{j}^\mu_V = (T^{-1}j'_V)^\mu, \tag{16f}$$

where $\boldsymbol{L} = \boldsymbol{r}'\times -i\nabla'$ is the orbital angular momentum, and $\boldsymbol{S} = (S_x, S_y, S_z)$ is the O(3) generator for ω-mesons with spin 1:

$$S_x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad S_y = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, \quad S_z = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{17}$$

The above results can also be derived from the Lagrangian density in the laboratory frame by the following replacements:

$$\psi = \psi', \quad \sigma\to\sigma', \quad V_\mu\to\tilde{V}'_\mu, \quad j^\mu_V\to\tilde{j}^\mu_V, \quad \gamma\to\gamma', \quad B\to B', \quad \partial_\mu\to D'_\mu = \partial'_\mu + \Gamma'_\mu = \partial'_\mu + \omega^{ij}_{,\mu}G_{ij}, \tag{18a}$$

where $\omega^{ij}_{,\mu}$ is the spin connection defined by

$$\omega^{ij}_{,0} = -i\epsilon_{ijk}\Omega_k, \quad \omega^{ij}_{,k} = 0, \tag{18b}$$

and G_{ij} is the spin matrix satisfied by

$$G_{ij}\sigma' = 0, \quad G_{ij}\psi' = \tfrac{1}{2}\gamma_{ij}\psi', \quad (G_{ij}\tilde{V})_k = \eta_{ik}\tilde{V}_j - \eta_{jk}\tilde{V}_i. \tag{18c}$$

Following the same procedure as that in the laboratory frame, the canonical conjugate fields are given as

$$\Pi' = i\psi'^\dagger, \quad \Pi'_\sigma = \dot{\sigma}' - i\boldsymbol{\Omega}\cdot\boldsymbol{L}\sigma', \quad \tilde{\Pi}_0 = B', \quad \tilde{\Pi}_k = F'_{0k} - i\boldsymbol{\Omega}\cdot(\boldsymbol{L}+\boldsymbol{S})\tilde{V}_k. \tag{19}$$

Then the Hamiltonian density is expressed as

$$\mathscr{H}' = \mathscr{H} - \boldsymbol{\Omega}\cdot\{\psi'^\dagger(\boldsymbol{L}+\boldsymbol{s})\psi' - i\Pi'_\sigma\boldsymbol{L}\sigma' - i[\tilde{\Pi}_0\boldsymbol{L}\tilde{V}_0 + \tilde{\Pi}_k(\delta_{kl}\boldsymbol{L} + \boldsymbol{S}_{kl})\tilde{V}_l]\}. \tag{20}$$

Thus, we described the relativistic many-body system in a three-dimensional rotating frame. We will next define the intrinsic frame in a triaxial deformed system.

The Hamiltonian H' in such a rotating frame is written as

$$H' = H - \boldsymbol{\Omega}\cdot\boldsymbol{J}, \quad \boldsymbol{J} = \boldsymbol{J}^{\mathrm{D}} + \boldsymbol{L}^\sigma + \boldsymbol{J}^\omega, \tag{21a,b}$$

where the Hamiltonian H is given as

$$H' = \int \mathscr{H}'\,d^3x', \quad H = \int \mathscr{H}\,d^3x', \tag{21c}$$

and $\boldsymbol{J}^{\mathrm{D}}$, $\boldsymbol{L}^\sigma$, and $\boldsymbol{J}^\omega$ are the angular momentum of the spinor field, of the scalar σ-meson, and of the vector ω-meson, respectively:

$$\boldsymbol{J}^{\mathrm{D}} = \int \psi'^\dagger(\boldsymbol{L}+\boldsymbol{S})\psi'\,d^3x', \quad \boldsymbol{L}^\sigma = -i\int \Pi'_\sigma\boldsymbol{L}\sigma'\,d^3x', \quad \boldsymbol{J}^\omega = -i\int [\tilde{\Pi}_0\boldsymbol{L}\tilde{V}_0 + \tilde{\Pi}_k(\delta_{kl}\boldsymbol{L} + \boldsymbol{S}_{kl})\tilde{V}_l]\,d^3x'. \tag{21d}$$

Then the equations of motion (16a)–(16e) in the rotating frame are rewritten by the canonical form

$$\dot{q}'_i = [q'_i, H']_{\mathrm{P}}, \quad \dot{\Pi}'_i = [\Pi'_i, H']_{\mathrm{P}}, \tag{22a}$$

where q'_i and Π'_i are defined as

$$q'_i = (\psi', \sigma', \tilde{V}_0, \tilde{V}_k), \quad \Pi'_i = (\Pi', \Pi'_\sigma, \tilde{\Pi}_0, \tilde{\Pi}'_k). \tag{22b}$$

Here the Poisson bracket $[F, G]_{\mathrm{P}}$ is defined as

$$[F,G]_{\rm P}=\int\left(\frac{\delta F}{\delta q'_i}\frac{\delta G}{\delta \Pi'_i}-(-)^{|i|}\frac{\delta F}{\delta \Pi'_i}\frac{\delta G}{\delta q'_i}\right)\mathrm{d}^3x'+\left(\frac{\partial F}{\partial\theta_k}\frac{\partial G}{\partial P_k}-\frac{\partial F}{\partial P_k}\frac{\partial G}{\partial\theta_k}\right), \tag{22c}$$

where $\delta F/\delta q'_i$ and $\delta F/\delta \Pi'_i$ are the functional derivative defined by

$$\frac{\delta F}{\delta q'_i}=\frac{\partial}{\partial q'_i}-\partial'_k\left(\frac{\partial F}{\partial(\partial'_k q'_i)}\right). \tag{22d}$$

The phase factor $(-)^{|i|}$ means that we use the minus $(-)$ for an odd number and the plus $(+)$ for an even number in the Grassmann variable q'_i. P_k is the conjugate momentum of the Euler angles θ_k: $[\theta_k, P_l]_{\rm P}=\mathrm{i}\delta_{kl}$. Since the Hamiltonian is rotationally invariant, the physical results do not depend on the choice of the rotating frame. This implies that the gauge invariance corresponds to the SO(3) symmetry. Thus, we need gauge-fixing conditions that determine the intrinsic frame. We will impose the constraints

$$\alpha_k\approx 0 \quad (k=1,2,3), \tag{23a}$$

satisfying the conditions

$$\mathrm{Det}([J_k,\alpha_l]_{\rm P})\neq 0, \quad [\alpha_k,\alpha_l]_{\rm P}=0. \tag{23b,c}$$

Since the Poisson bracket must be worked out before we make use of the constraint equations, we use a different equality sign $\approx$ from the usual $=$. Consequently, we call these equations (23a) weak equations. We cannot uniquely determine the α_k satisfying the conditions (23b) and (23c). At this point, we will choose the following constraints to determine the principal axes (PA) frame:

$$\alpha_x=Q_{22}-Q_{2\,-2}, \quad \alpha_y=Q_{21}+Q_{2\,-1}, \quad \alpha_z=Q_{21}-Q_{2\,-1}, \tag{24a}$$

where Q_{2M} are the quadrupole tensors

$$Q_{2M}=\int \mathrm{d}^3x'\,\psi'^{\dagger}r'^2Y_{2M}\psi'+\int \mathrm{d}^3x'\,\Pi'_\sigma r'^2Y_{2M}\sigma'+\int \mathrm{d}^3x'\,\tilde{\Pi}_\mu r'^2Y_{2M}\tilde{V}_\mu. \tag{24b}$$

The consistency conditions for arbitrary time are

$$\mathrm{i}\dot{\alpha}_k=[\alpha_k,H']_{\rm P}=[\alpha_k,H]_{\rm P}-\Omega_l[\alpha_k,J_l]_{\rm P}=0. \tag{25}$$

From these conditions, the angular velocities Ω_k are determined as

$$\Omega_k=-[H,\alpha_l]_{\rm P}\Phi^{-1}_{lk}, \tag{26a}$$

where Φ^{-1}_{lk} are the inverse matrix elements of $[\alpha_k, J_l]_{\rm P}$ given by

$$[\alpha_k,J_l]_{\rm P}\Phi^{-1}_{lk'}=\delta_{kk'}. \tag{26b}$$

Inserting (8a) into (26a), we obtain the relationship

$$V_{ki}\dot{\theta}_i=-[H,\alpha_l]_{\rm P}\Phi^{-1}_{lk}. \tag{27}$$

These differential equations give the connection between the Euler angles θ_i and the variables (q'_i, Π'_i). Upon solving the differential equations (27), one finds that the Euler angles θ_i are expressed by the variables (q'_i, Π'_i). The Hamiltonian H' of eq. (21a) then satisfies the consistency conditions. However, eq. (25) admits solutions for which $\alpha_k\neq 0$. Such solutions involve the admixture of spurious modes. In order to eliminate the spurious mode, for an arbitrary physical quantity F we define $\tilde{F}$ as follows:

$$\tilde{F}=F+[F,\alpha_l]_{\rm P}\Phi^{-1}_{lk}\chi_k+[F,\chi_l]_{\rm P}\Psi^{-1}_{lk}\alpha_k, \quad \chi_k=J_k-I_k, \tag{28a,b}$$

where I_k are the collective version of the angular momentum referred to in the intrinsic frame, and Ψ^{-1}_{lk} are the inverse matrix elements of $[J_k, \alpha_l]_{\rm P}$ given by

$$[J_k, \alpha_l]_P \Psi^{-1}_{lk'} = \delta_{kk'} , \tag{28c}$$

and $\tilde{F}$ is the invariant part satisfying the relationship

$$[\tilde{F}, \alpha_k]_P = 0 . \tag{29}$$

Putting $F=\alpha_k$, the $\tilde{\alpha}_k$ satisfy $\tilde{\alpha}_k = 0$ as the strong equality. The α_k and χ_k are second-class constraints. It is now convenient to introduce the Dirac bracket defined as

$$[F, G]_D = [F, G]_P + [F, \alpha_k]_P \Phi^{-1}_{kl} [\chi_l, G]_P + [F, \chi_k]_P \Psi^{-1}_{kl} [\alpha_l, G]_P . \tag{30}$$

Then the Dirac brackets of the variables (q_i, Π_i) become

$$[q'_i(x_0, \boldsymbol{x}), \Pi'_j(x_0, \boldsymbol{y})]_D = \delta_{ij}\delta(\boldsymbol{x}-\boldsymbol{y}) + [q'_i, \alpha_k]_P \Phi^{-1}_{kl} [J_l, \Pi'_j]_P + [q'_i, J_k]_P \Psi^{-1}_{kl} [\alpha_l, \Pi'_j]_P , \tag{31a}$$

$$[q'_i(x_0, \boldsymbol{x}), q'_j(x_0, \boldsymbol{y})]_D = [q'_i, \alpha_k]_P \Phi^{-1}_{kl} [J_l, q'_j]_P + [q'_i, J_k]_P \Psi^{-1}_{kl} [\alpha_l, q'_j]_P , \tag{31b}$$

$$[\Pi'_i(x_0, \boldsymbol{x}), \Pi'_j(x_0, \boldsymbol{y})]_D = [\Pi'_i, \alpha_k]_P \Phi^{-1}_{kl} [J_l, \Pi'_j]_P + [\Pi'_i, J_k]_P \Psi^{-1}_{kl} [\alpha_l, \Pi'_j]_P . \tag{31c}$$

The Dirac brackets of the angular momentum are

$$[J_k, J_l]_D = -\mathrm{i}\epsilon_{klm} J_m . \tag{32}$$

Let us next perform the canonical quantization with constraints. Following the procedure of the Dirac quantization [15], the quantization is carried out by the replacements

$$[\ ,\ ]_D \rightarrow [\ ,\ ] , \quad q'_i \rightarrow \hat{q}'_i , \quad \Pi'_i \rightarrow \hat{\Pi}'_i , \qquad J_k \rightarrow \hat{J}_k , \quad \alpha_k \rightarrow \hat{\alpha}_k , \tag{33a,b}$$

where $[\hat{F}, \hat{G}]$ means the commutation relation for the boson operator and the anti-commutation relation for the fermion operator. Then eqs. (30)–(32) become

$$[\hat{F}, \hat{G}] = [\hat{F}, \hat{G}]_P + [\hat{F}, \hat{\alpha}_k]_P \hat{\Phi}^{-1}_{kl} [\hat{\chi}_l, \hat{G}]_P + [\hat{F}, \hat{\chi}_k]_P \hat{\Psi}^{-1}_{kl} [\hat{\alpha}_l, \hat{G}]_P , \tag{34a}$$

$$[\hat{q}'_i(x_0, \boldsymbol{x}), \hat{\Pi}'_j(x_0, \boldsymbol{y})] = \delta_{ij}\delta(\boldsymbol{x}-\boldsymbol{y}) + [\hat{q}'_i, \hat{\alpha}_k]_P \hat{\Phi}^{-1}_{kl} [\hat{J}_l, \hat{\Pi}'_j]_P + [\hat{q}'_i, \hat{J}_k]_P \hat{\Psi}^{-1}_{kl} [\hat{\alpha}_l, \hat{\Pi}'_j]_P , \tag{34b}$$

$$[\hat{q}'_i(x_0, \boldsymbol{x}), \hat{q}'_j(x_0, \boldsymbol{y})] = [\hat{q}'_i, \hat{\alpha}_k]_P \hat{\Phi}^{-1}_{kl} [\hat{J}_l, \hat{q}'_j]_P + [\hat{q}'_i, \hat{J}_k]_P \hat{\Psi}^{-1}_{kl} [\hat{\alpha}_l, \hat{q}'_j]_P , \tag{34c}$$

$$[\hat{\Pi}'_i(x_0, \boldsymbol{x}), \hat{\Pi}'_j(x_0, \boldsymbol{y})] = [\hat{\Pi}'_i, \hat{\alpha}_k]_P \hat{\Phi}^{-1}_{kl} [\hat{J}_l, \hat{\Pi}'_j]_P + [\hat{\Pi}'_i, \hat{J}_k]_P \hat{\Psi}^{-1}_{kl} [\hat{\alpha}_l, \hat{\Pi}'_j]_P . \tag{34d}$$

The Dirac brackets of the angular momentum are

$$[\hat{J}_k, \hat{J}_l] = -\mathrm{i}\epsilon_{klm} \hat{J}_m , \tag{35}$$

where $[\hat{F}, \hat{G}]_P$ means the operator that is obtained by the replacements (33a), (33b) after working out the Poisson bracket. From eqs. (34b)–(34d), it is clear that $\hat{q}'_i$ and $\hat{\Pi}'_i$ are not fermions or bosons. The commutation relations (34b)–(34d) contain the deviations from the fermion and boson rules. Putting $\hat{F}=\hat{q}'_i$ and $\hat{G}=\hat{\alpha}_k$in eq. (34a), it is easily found that $[\hat{q}'_i, \hat{\alpha}_k] = 0$. This means that the $\hat{\alpha}_k$ play the role of constants of motion due to the constraints. The angular momentum algebra (35) obeys exactly *the minus-sign rules* of the usual commutation relations which are well known as the commutation rules with respect to the rotating body-fixed frame. This is due to the non-bosonic commutation relations (34b)–(34d).

In conclusion, we have presented a canonical formulation of three-dimensional rotation and intrinsic motion in a relativistic many-body system. In the moving frame, the technique is analogous to that of general relativity. Since the covariance of the Lagrangian was needed, we obtained the Lagrangian in the moving frame and derived the equations of motion. The intrinsic frame was determined by imposing constraints. Then, the motions in the relativistic many-body system were completely separated into three-dimensional collective rotation and the intrinsic motion. It would be interesting to use a random phase approximation in our formulation, and to compare it with the RPA in a non-relativistic formulation by Marshalek. This investigation is now in progress.

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