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SHELL-FILLING DEPENDENT EFFECTS OF WOBBLING MOTION ON SIGNATURE INVERSION IN ODD-A NUCLEI

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Abstract:

Shell-filling dependent effects of the wobbling motion on the signature inversion in odd-A nuclei are studied by means of the quasiparticle-vibration coupling model in the rotating frame. The inversion in the negative-gamma three-quasiparticle bands is shown to be driven by the coupling with the wobbling, while that in the positive-gamma bands is brought about by the static deformation as has been explained in the previous works.

Signature inversion, a phenomenon that a $\Delta I = 1$ rotational band decouples into two signature sequences invertedly, has been one of the interesting issues in the study of rotating nuclei. The inversion in the $(\pi h_{11/2})^1(\nu i_{13/2})^2$ bands has been observed systematically in the $A \simeq 160$ region and explained in terms of the static triaxial deformation ($\gamma > 0$) as in the case of the odd-odd nuclei[1]. The inversion was observed[2], however, also in some nuclei in which $\gamma < 0$ is expected according to the shell filling. This indicates that the theory should be extended in some way; for example, to incorporate the fluctuations, related to the gamma degree of freedom, around the mean field: the shape fluctuation (gamma vibration) and the fluctuation of the rotational axis (wobbling motion) in triaxial nuclei.

We have studied this problem by means of the quasiparticle-vibration coupling (QVC) model, that is an extension of the cranking model, analytically and numerically. The wobbling and gamma-vibrational phonons are constructed by means of the RPA in the rotating frame. The lowest energy RPA solution with signature $r = -1$ can be called the wobbling in general. This solution reduces to a gamma vibration with $K = 2$, where K is the angular-momentum projection onto the z-axis, in the limit of vanishing rotational frequency $\omega_{\rm rot}$ and static triaxial deformation γ, while it becomes a "true" wobbling-like mode with strongly mixed K in triaxial nuclei at high spins. Shimizu and Matsuyanagi showed that both the gamma vibration with $r = +1$ and the wobbling, which they called the gamma vibration with $r = -1$, become less collective and more K-mixed after the g-s band crossing, and that the decrease in the collectivity was more remarkable in the former[3]. Their discussion was confined to the absolute values of the (doubly-stretched, which is indicated by the tildes) phonon matrix elements (ground-state expectation values), $\widetilde{T}_K^{(\pm)} =< [\widetilde{Q}_K^{(\pm)}, X^\dagger_{(\pm)}] >$, associated with the phonon $X^\dagger_{(\pm)}$ with $r = \pm 1$. Besides, their relative sign was shown to reflect the sign of the equilibrium γ [4]:

$$\frac{\widetilde{T}_2^{(-)}}{\widetilde{T}_1^{(-)}} = f(\omega_{\rm wob})\frac{\sin \gamma_{\rm eq}}{\sin(\gamma_{\rm eq}+60^\circ)} \quad , \tag{1}$$

with $f(\omega_{\rm wob})$ being negative-definite. In addition to the fact that this relative sign determines the spin dependence of the $B(E2:\text{wob}\rightarrow\text{yrast})$, it governs the sign of signature splitting in the adjacent odd-A nuclei as shown later. We, therefore, discuss a quantity $\widetilde{T}_2^{(-)}/\widetilde{T}_1^{(-)}$ briefly here before proceeding to the QVC.

The calculated neutron-number dependence of the ratio $\widetilde{T}_2^{(-)}/\widetilde{T}_1^{(-)}$ is shown in the upper part of fig.1 both for the g-band and s-band configurations. In the g-band case, the K-mixing is weak, in other words the character of the solution is still gamma-vibrational, and almost independent of N. In contrast, the K-mixing is strong in the s-band case. In addition, the ratio changes its sign around $N = 92$. This can be interpreted to be related to the shape driving force of the aligned two neutrons[4] even when the shape of the potential is set to be axially symmetric artificially.

The $r = -1$ QVC vertices are given by

$$\begin{aligned}\Lambda(\text{wob}\otimes \text{f},\text{u}) &= -(\kappa_1^{(-)}\widetilde{T}_1^{(-)}\langle \text{f}|\,\widetilde{Q}_1^{(-)}\,|\text{u}\rangle + \kappa_2^{(-)}\widetilde{T}_2^{(-)}\langle \text{f}|\,\widetilde{Q}_2^{(-)}\,|\text{u}\rangle) \quad ,\\ \Lambda(\text{wob}\otimes \text{u},\text{f}) &= -(\kappa_1^{(-)}\widetilde{T}_1^{(-)}\langle \text{f}|\,\widetilde{Q}_1^{(-)}\,|\text{u}\rangle - \kappa_2^{(-)}\widetilde{T}_2^{(-)}\langle \text{f}|\,\widetilde{Q}_2^{(-)}\,|\text{u}\rangle) \quad ,\end{aligned} \tag{2}$$

where f and u denote favored and unfavored states, respectively, $\kappa_K^{(-)}$'s are the strengths of the doubly-stretched Q-Q interaction, and $\langle \text{f}|\,\widetilde{Q}_K^{(-)}\,|\text{u}\rangle$'s are the quasi-particle matrix elements. The expressions for the $r = +1$ sector are similar. Once these vertices are given, the QVC Hamiltonian can easily be diagonalized. But before going further to the final results, we examine the expressions for the energy shifts due to the QVC within the second-order perturbation. They are given by

$$\begin{aligned}\delta e'_{\rm u} &= \sum_{\rm f'}\frac{|\Lambda(\text{wob}\otimes \text{f}',\text{u})|^2}{e'_{\rm u}-e'_{\rm f'}-\hbar\omega_{\rm wob}} + \sum_{\rm u'}\frac{|\Lambda(\gamma(+)\otimes \text{u}',\text{u})|^2}{e'_{\rm u}-e'_{\rm u'}-\hbar\omega_{\gamma(+)}} \quad ,\\ \delta e'_{\rm f} &= \sum_{\rm u'}\frac{|\Lambda(\text{wob}\otimes \text{u}',\text{f})|^2}{e'_{\rm f}-e'_{\rm u'}-\hbar\omega_{\rm wob}} + \sum_{\rm f'}\frac{|\Lambda(\gamma(+)\otimes \text{f}',\text{f})|^2}{e'_{\rm f}-e'_{\rm f'}-\hbar\omega_{\gamma(+)}} \quad .\end{aligned} \tag{3}$$

The difference between them gives the additional signature splitting due to the QVC. Looking at the first terms in these expressions closely, the additional signature splitting stemming from the $r = -1$ phonon, which we call $\delta(\Delta e')_{(-)}$

hereafter, is almost zero if the phonon is not K-mixed, namely, one of $\widetilde{T}_K^{(-)}$'s in eq.(2) is zero, because the difference in the denominators is negligible.

The next interest is whether the QVC enhances the normal signature splitting or produces signature inversion. As for the signature-partner vertices, which become dominant at high spins, the single quasiparticle matrix elements, $\langle \text{f}|\,\widetilde{Q}_K^{(-)}\,|\text{u}\rangle$'s, have the same sign in most high-j cases[5]. Combining this with eq.(1) for the phonon, we obtain

$$|\Lambda(\text{wob}\otimes\text{f},\text{u})| \gtrless |\Lambda(\text{wob}\otimes\text{u},\text{f})| \qquad \text{for} \qquad \gamma \lessgtr 0 \quad . \tag{4}$$

This results in a selection rule that the wobbling components mix strongly into the unfavored (favored) state and consequently pushes $e'_{\rm u}$ ($e'_{\rm f}$) down, namely, signature inversion is produced (reduced) due to the QVC in $\gamma < 0$ ($\gamma > 0$) cases. See fig.2. As for the $r = +1$ contributions, that is, the second terms in eq.(3), they enhance the normal signature splitting in the odd-Z nuclei with high proton shell-fillings λ_π in the $h_{11/2}$ shell, because $\langle \text{f}|\,\widetilde{Q}_2^{(+)}\,|\text{f}\rangle > \langle \text{u}|\,\widetilde{Q}_2^{(+)}\,|\text{u}\rangle$ in these nuclei[5] for which experimental information is available. These partial contributions are compiled in table 1.

An example of the calculated $\delta(\Delta e')_{(\pm)}$ is presented in the lower part of fig.1. In the 1qp case, $\delta(\Delta e')_{(+)}$ and $\delta(\Delta e')_{(-)}$ almost cancel each other because the character of the wobbling is gamma-vibrational before the g-s band crossing as mentioned above. Therefore the sign of signature splitting is determined by the static γ deformation. This is the reason why signature inversion has never been observed in high-j 1qp bands. In contrast, the $r = +1$ gamma vibration loses its collectivity after the g-s band crossing drastically, in consequence the wobbling contribution can compete with that from the static γ deformation, especially at the high-λ_ν (in the $i_{13/2}$ shell) nuclei.

Now, we proceed to the results of more realistic calculations with non-zero γ deformations. The result for the 3qp band of the ^{166}Yb plus $(\pi_{11/2})^1$ system, which is the average of ^{165}Tm and ^{167}Lu, is shown in the upper part of fig.3. In the cranking calculation, represented by the broken line, the sign of signature splitting is positive because of the negative-γ deformation. The QVC results are represented by the dot-dashed and solid lines. The former and latter are the results of the second-order perturbation including one-phonon states, and the diagonalization including one- and two-phonon states, respectively. They produce signature inversion and reproduce the data for Lu [2] well. The result for the 3qp band of the ^{160}Yb plus $(\pi_{11/2})^1$ system is shown in the upper part of fig.4. In this case, the cranking model gives signature inversion because of the positive-γ deformation. Although the QVC contributions reduce the magnitude of the inversion as expected analytically in the above, the inversion survives finally. In the lower parts of figs. 3 and 4, the corresponding results for the $B(M1)$ values are shown. They are quite consistent with those for the Routhians in the upper parts qualitatively. An interesting feature of them is that the inversion disappears in the QVC result for the positive-γ case. One reason for this feature is the fact that the frequency region where the inversion occurs is narrower for the $B(M1)$ than for the energy already in the cranking calculation[7].

The present study has clarified that the QVC, especially with the $r = -1$ phonon, produces the shell-filling-dependent signature dependence in the quasi-particle energy as in the intraband $B(E2 : I \rightarrow I-1)$ [5]. The difference is that $\Delta e'$ depends on the shell-filling of the aligned neutrons and the origin of this shell-filling dependence is the relative sign between $\widetilde{T}_1^{(-)}$ and $\widetilde{T}_2^{(-)}$, while $\Delta B(E2 : I \rightarrow I-1)$ depends on the shell-filling of the odd proton and the origin is the relative sign between $\langle f|\widetilde{Q}_2^{(-)}|u\rangle$ and $\langle f|J_z|u\rangle$. In other words, the shell-filling dependence of the former reflects the rotational K-mixing in the $r = -1$ phonon while that of the latter reflects the $K = 2$ (gamma-vibrational) collectivity associated with the phonon. In comparison with the effects brought about by the static γ deformation in the cranking model, the QVC changes the signature dependence qualitatively in the $\gamma < 0$ cases, whereas it gives small corrections in the $\gamma > 0$ cases, both in $\Delta e'$ and $\Delta B(E2 : I \rightarrow I-1)$.

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Table 1. Partial contributions to signature splitting in the odd-Z nuclei with $A \simeq 160$ from the static triaxial deformation, the wobbling motion ($r = -1$) and the gamma vibration with ($r = +1$); n and i denote contributions to the normal ($\Delta e' = e'_u - e'_f > 0$) and inverted ($\Delta e' < 0$) splittings, respectively. (From ref.6).)

band		γ_{eq}	static	wobbling	$\gamma(+)$-vib.
$(\pi h_{11/2})^1$		$-$	n	i	n
$(\pi h_{11/2})^1(\nu i_{13/2})^2$,	$\lambda_\nu \gtrsim \epsilon_{3/2}$	$-$	n	i	n
$(\pi h_{11/2})^1(\nu i_{13/2})^2$,	$\lambda_\nu < \epsilon_{3/2}$	$+$	i	n	n

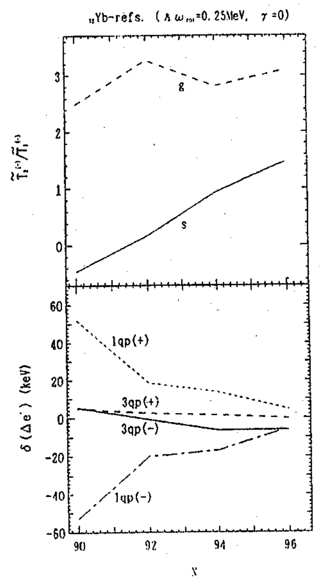


Fig.1 Ratios of the phonon matrix elements of the $r = -1$ doubly-stretched quadrupole operators as a function of the neutron number calculated for the Yb_{even} isotopes at $\hbar\omega_{rot} = 0.25$MeV with $\gamma^{(pot)} = 0$. Adopted $\beta^{(pot)}$'s are 0.20, 0.23, 0.25 and 0.27 for N =90, 92, 94 and 96, respectively. The broken and solid lines represent the results for the g-band and s-band configurations, respectively (upper part). Partial contributions to signature splitting from the coupling with the $r = +1$ and -1 phonons, calculated by treating the quasiparticle-vibration coupling within the second-order perturbation, both for the yrast 1qp and 3qp bands of the Yb_{even} plus $(\pi h_{11/2})^1$ systems (lower part).

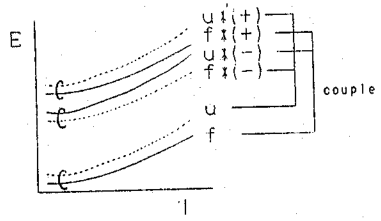
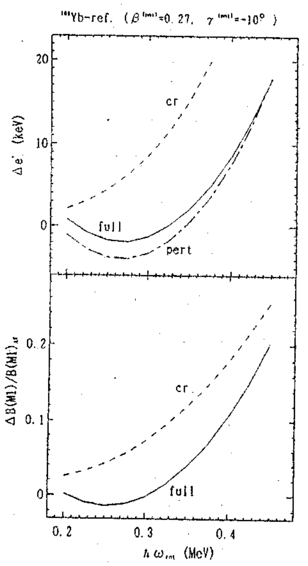


Fig.2 A schematic drawing of the quasiparticle-vibration coupling considered in the present study.



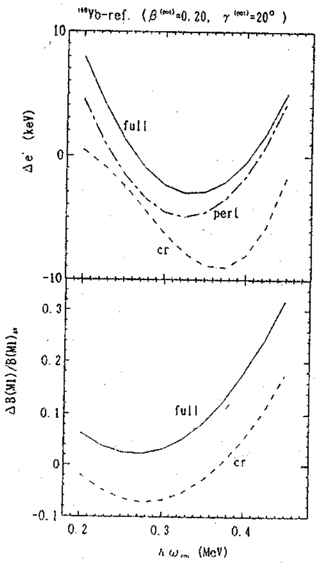


Fig.3 Calculated signature splittings of the yrast $(\pi h_{11/2})^1(\nu i_{13/2})^2$ band in the ^{166}Yb plus $(\pi h_{11/2})^1$ system as a function of the rotational frequency. "Cr" denotes the result of the cranking calculation, while "pert" and "full" denote those of the quasiparticle-vibration coupling (QVC) calculations within the second-order perturbation and diagonalization, respectively. The QVC results were smoothed parabolically in order to remove accidental losses of collectivity in the RPA phonons due to approaches of single-particle like RPA solutions. Parameters used are $\beta^{(\mathrm{pot})} = 0.27$, $\gamma^{(\mathrm{pot})} = -10°$, $\Delta_p = 1.10$ MeV and $\Delta_n = 0.73$ MeV (upper part, taken from ref.6)). Calculated signature dependence of the $B(M1)$ values:
$\Delta B(M1)/B(M1)_{\mathrm{AV}} = 2(B(M1:\mathrm{f}\rightarrow\mathrm{u}) - B(M1:\mathrm{u}\rightarrow\mathrm{f}))/(B(M1:\mathrm{f}\rightarrow\mathrm{u}) + B(M1:\mathrm{u}\rightarrow\mathrm{f}))$.
These values are almost independent of the adopted g-factors (lower part).

Fig.4 The same as fig.3 but for the ^{160}Yb plus $(\pi h_{11/2})^1$ system. Parameters used are $\beta^{(\mathrm{pot})} = 0.20$, $\gamma^{(\mathrm{pot})} = 20°$, $\Delta_p = 1.24$ MeV and $\Delta_n = 0.75$ MeV.