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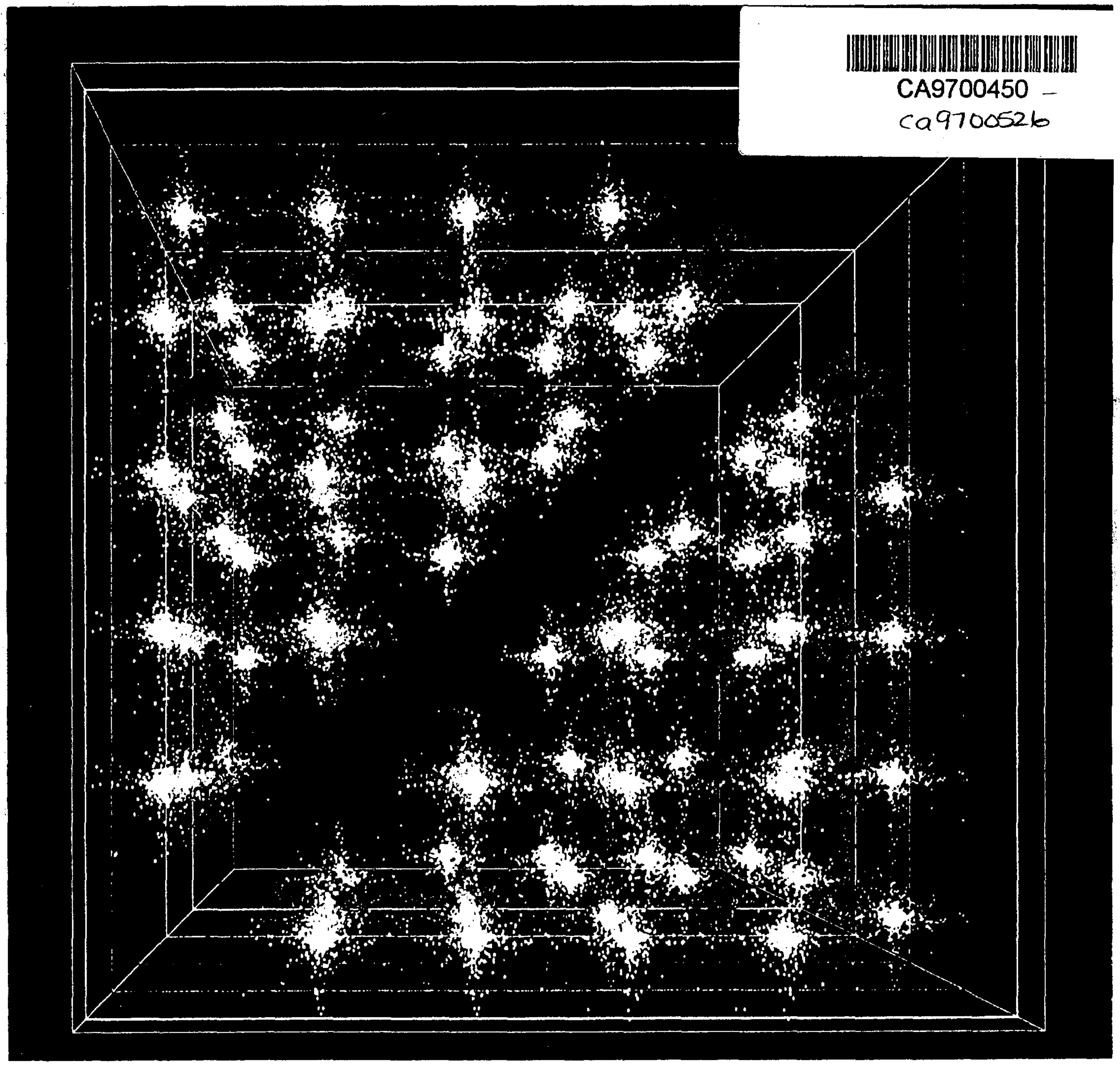

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NUCLEAR WOBBLING MOTION AND PROPERTIES OF E2-TRANSITIONS

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Abstract

The nuclear wobbling motion associated with the static triaxial deformation are discussed based on a microscopic theory. Properties of the $E2$-transitions between the one-phonon wobbling band and the yrast (vacuum) band are studied and their characteristic features are suggested.

1. Nuclear Wobbling Motion and E2-transition

The nuclear wobbling motion is a new elementary mode of excitations which is predicted to appear at high-spin states[1] by analogy with the classical motion of the asymmetric top. The lowest energy motion of the classical top is a uniform rotation around a certain principal axis, which corresponds to the yrast states, and the excited motion is such that the angular momentum vector in the body-fixed frame fluctuates, i.e., "wobbles" or "precesses", around the main rotation axis. Note that the static triaxial deformation is necessary for such a three-dimensional non-uniform rotation to be realized. Recent measurements of detailed properties of the electromagnetic(EM) transitions rates revealed that the triaxial degrees of freedom, either of static or of dynamic (vibrational) nature, do play an important role at high-spin states. It is, therefore, interesting to ask how it appears at this stage.

A band structure expected for *the triaxial rigid-rotor* at high-spins is shown schematically in Fig.1. States are classified into two sequences, the "horizontal" and "vertical" excitations, and their slopes are connected to the largest and the smallest moment of inertia, respectively. Members of the horizontal sequence are connected by the stretched ($\Delta I = \pm 2$) $E2$-transitions which is characteristic to the usual collective rotation around the axis of largest moment of inertia. The vertical sequence is generated by superimposing the rotation around the axes other than the main rotation and can be regarded as multipole excitations of the $\Delta I = \pm 1$ (signature $\alpha = 1$) "wobbling phonon".

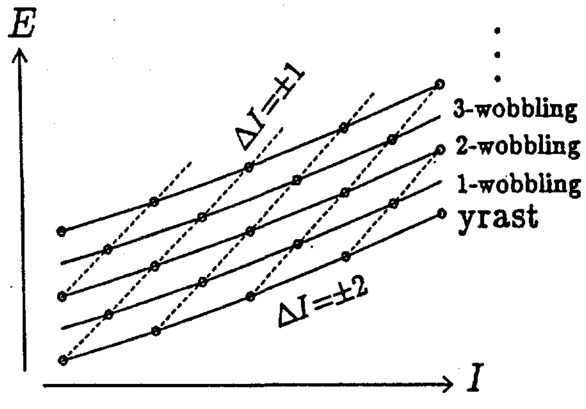
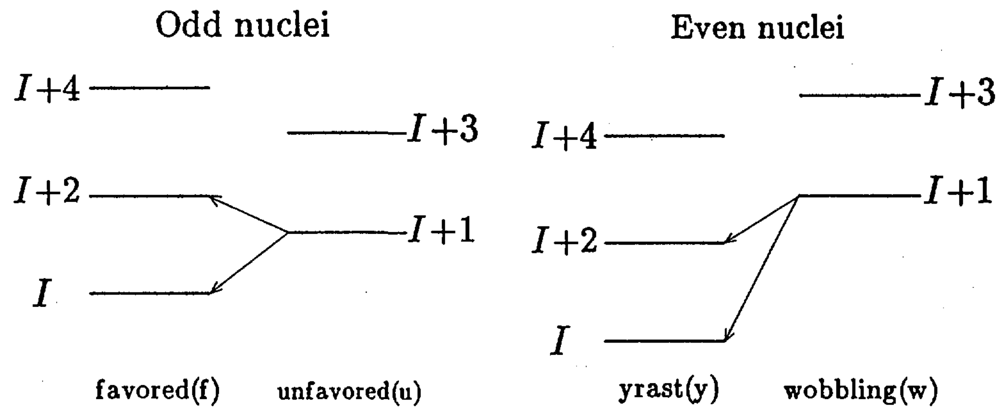


Fig.1. A schematic figure of the band structure expected from the triaxial rigid-rotor.

Study of such a genuine three-dimensional rotational motion is important because it is related to fundamental questions: In what extent the nucleus behaves as a "rotor"?, or how the "intrinsic" (body-fixed) frame is defined? The definition is highly non-trivial in the case of general rotations around three spatial axes since the underlying nucleonic motions and, therefore, the selfconsistent deformation are strongly affected by the rotational motion in the many-body system like atomic nuclei. The dynamical fluctuations of the angular momentum vector in the intrinsic frame is, in general, a large amplitude motion. Such a general case has been studied in Refs.[2,3] by using the time-dependent variational method with a proper definition of the intrinsic frame. Unfortunately the results of Refs.[2,3] are rather complicated and it is difficult to understand the essential property of the wobbling motion. In the yrast region such a genuine three-dimensional rotational motion is expected to be of small amplitude, and a fully microscopic treatment is possible within the random phase approximation (RPA).[4] Then the wobbling mode is described as a kind of "vibration", or the wobbling phonon, i.e., the quantized precession of the angular momentum vector. Here we discuss such a relatively simple case and concentrate on the 1-wobbling band in Fig.1.

Since various kinds of rotational bands, both of collective and of single-particle nature, are observed, it is difficult to identify the wobbling band among them only from the energy spectra. The information of the electromagnetic transitions, especially the $B(E2)$ which sensitively reflects the collective properties, is important in this respect. For the general band structure shown in Fig.1, the $B(E2)_{\Delta I=\pm 2}$ of the horizontal transitions are related to the static deformation around the main rotation-axis, or the cranking-axis (x-axis), as usual. The $B(E2)_{\Delta I=\pm 1}$ of the vertical transitions are determined by the electro-magnetic properties of the wobbling phonon and reflect the effect of dynamical fluctuations of the angular momentum vector.



$$B(M1, u \rightarrow f)_{\Delta I=\pm 1} \approx |<f|(\mu_z \pm i\mu_y)|u>|^2$$

$$B(E2, w \rightarrow y)_{\Delta I=\pm 1} \approx |<y|(Q_1^{(-)} \pm Q_2^{(-)})|w>|^2/2$$

Fig.2. An analogy between the $\Delta I = \pm 1$ $M1$-transitions for the quasiparticle band and the $\Delta I = \pm 1$ $E2$-transitions for the wobbling band.

Within the lowest order approximation in the $1/I$-expansion,[4] $E2$-transition rate is expressed as

$$B(E2; i \rightarrow f)_{\Delta I=\pm1} \approx |\langle f|(Q_1^{(-)} \pm Q_2^{(-)})|i\rangle|^2/2, \tag{1}$$

where $Q_K^{(-)}$ $(K = 1,2)$ is the signature-coupled quadrupole operator (quantized with respect to the z-axis). Now the analogy to the case of interband $M1$-transitions between the favoured and unfavoured quasiparticle bands in odd nuclei, are quite obvious: The effects of the rotation and the triaxial deformation make both the K=1 and 2 components of quadrupole transitions non-vanishing (K-mixing) and may cause characteristic staggering in $\Delta I = \pm1$ $E2$-transitions between the yrast and the wobbling band, see Fig.2.

2. A Microscopic Treatment

It is interesting to see how the rotor-picture appears from the microscopic viewpoint.[4] The starting point of the microscopic description of the wobbling motion is the cranked mean-field approximation followed by the time-dependent Hartree-Bogoliubov (TDHB) method in the uniform-rotating(UR) frame In the small amplitude limit (RPA) the signature $\alpha = 0$ (even-I transfer) and $\alpha = 1$ (odd-I transfer) modes decouple and the wobbling motion belongs to the latter.

Assuming the quadrupole-field as a main deformation component, the time-dependent single-particle hamiltonian in the presence of the wobbling motion is written as

$$h_{UR}(t) = h_{def} - \Omega J_x - \kappa_y \mathcal{Q}_y(t) Q_y - \kappa_z \mathcal{Q}_z(t) Q_z. \tag{2}$$

Here Ω is the cranking frequency, the Q_i $(i = y, z)$ is a non-diagonal component of the quadrupole tensor, $Q_i = \sum_{a=1}^{A}(x_j x_k)_a$ $(i, j, k$=cyclic), and $\mathcal{Q}_i(t)$ is its expectation value with respect to the corresponding UR-frame TDHB state. Note that the deformation fields in eq.(2), $Q_y \propto Q_{K=1}^{(-)}$ and $iQ_z \propto Q_{K=2}^{(-)}$ (cf. eq.(1)), so that they correspond to the dynamical fluctuation of the triaxiality. If $\mathcal{Q}_i(t) \ll \alpha_i$, with α_i being the static deformation, i.e., the expectation value of the diagonal component, $\alpha_i \equiv \langle \sum_{a=1}^{A}(x_j^2 - x_k^2)_a \rangle$, it is possible to make a time-dependent coordinate transformation to the body-fixed or the principal-axis(PA) frame which is introduced in such a way as the shape fluctuation disappears, $\mathcal{Q}_i^{PA}(t) = 0$.[2,4] Then the three-dimensional nature of the rotation shows up in this new coordinate frame and the TDHB hamiltonian now takes the form,

$$h_{PA}(t) = h_{def} - \Omega J_x - \Omega_y(t) J_y - \Omega_z(t) J_z.\text{eq.}(3)$$

Namely, the angular frequency vector, $\mathbf{\Omega}(t)$, or the angular momentum vector, $\langle \mathbf{J} \rangle_{PA}(t)$, wobbles around the cranking x-axis (note $\Omega_x(t) \approx \Omega$ and $\langle J_x \rangle_{PA}(t) \approx \langle J_x \rangle = I$ in the RPA order). After the quantization the time-dependences of the physical quantities can be expressed as linear combinations of those corresponding to the RPA normal-modes.

Then the moment of inertia around the axes perpendicular to the cranking axis, $\mathcal{J}_i^{eff} \equiv \langle J_i \rangle_{PA}^{(n)} / \Omega_i^{(n)}$ $(i = y, z)$, for each n-th wobbling phonon state can be calculated[4] full-microscopically without any macroscopic assumptions. Moreover, it can be shown that the RPA-eigen value equation (and also the expressions of E2-transition matrix elements) in the UR-frame is transformed to the well-known "wobbling equation"[1] in terms of there moments of inertia, $\mathcal{J}_x \equiv \langle J_x \rangle / \Omega$, $\mathcal{J}_y^{eff}$ and $\mathcal{J}_z^{eff}$ (and the static deformations, α_x, α_y and α_z). Note that any physical observables in the PA-frame can be obtained uniquely from the RPA-phonon amplitudes which are calculated in a usual manner in the UR-frame.

An example of calculations for the wobbling mode excited on top of the s-band of ^{182}Os are shown in Fig.3, in which staggering between $\Delta I = +1$ and $\Delta I = -1$ $B(E2)$ is apparent. Note that this characteristic feature of the transition amplitude are connected to the dynamical fluctuation of the angular momentum vector in the PA-frame, or $(\mathcal{J}_x, \mathcal{J}_y^{eff}, \mathcal{J}_z^{eff})$, as is discussed above. The microscopic calculation predicts[5] that these moments of inertia change from "axially symmetric like" to "triaxial like" even though the mean-field parameters are kept constant. Furthermore, the predicted dependence of these inertia on the triaxiality parameter is quite different from one expected in the macroscopic models like the rigid-body or the irrotational-flow.[6]

3. Discussions

In this conference an interesting possibility of the "tilted cranking" bands is discussed,[7] where the direction of the angular momentum vector in the intrinsic frame deviates from the

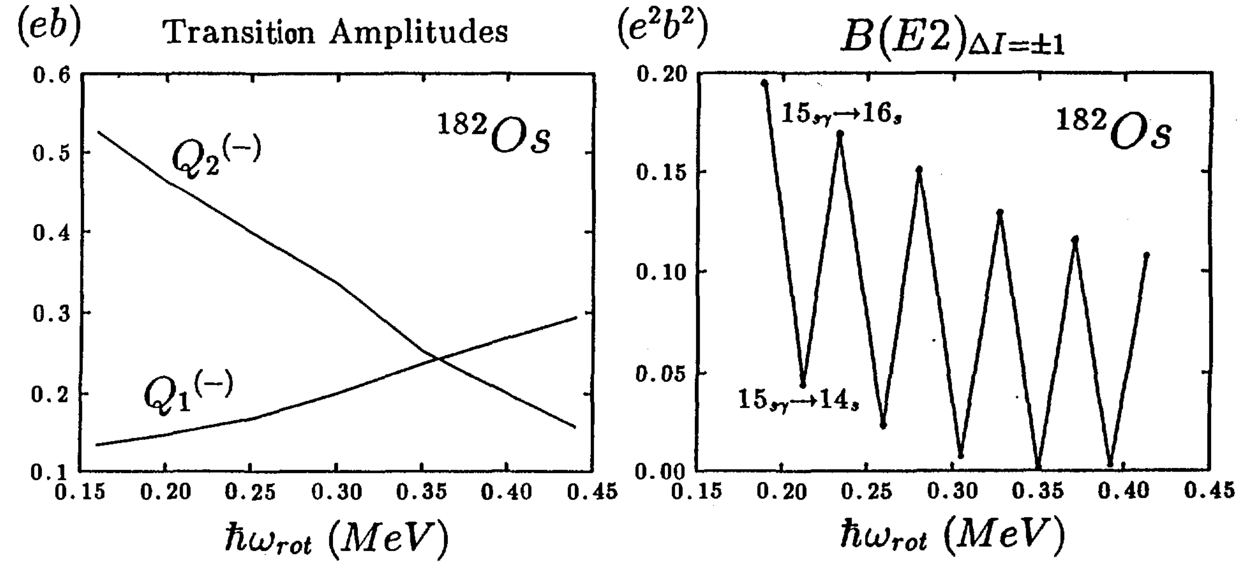


Fig.3. An example of the microscopic RPA calculation for the wobbling mode excited on top of *the s-band* of ^{182}Os. The transition amplitudes (left panel) and $B(E2)_{\Delta I=\pm1}$ (right panel) are shown as functions of the rotational frequency. The calculational procedure is the same as Ref.[5] but with using the RPA equation where the Nambu-Goldstone mode is explicitly decoupled.

principal axes but still the rotation is uniform, i.e., $\Omega \parallel \langle \mathbf{J} \rangle_{PA}$. Such a motion is not allowed in the classical-top, thus, is not the same thing as the wobbling motion. It is, however, similar to the wobbling in the sense that both reflect the three-dimensional nature of the nuclear rotational motion. In fact the strong coupling bands based on the high-K isomers systematically observed in the $Hf-W$ region can be regard as a special case of the wobbling motion, called the "precession",[8] i.e., the case where only the vertical sequences in Fig.1 are present (the horizontal sequences correspond to the rotation around the symmetry-axis and are forbidden in the quantal systems).

At least up to now, there are no definite experimental data which indicate the existence of the nuclear wobbling motions. In order to identify the wobbling band we suggest to look for the continuation of odd-I sequence ($\alpha = 1$) of the γ-band after the back-bending, i.e., excited on top of the s-band where sizable static triaxiality is sometimes predicted, in even-even nuclei according to our theoretical calculations.[5,9] This is because the wobbling mode is closely related to the dynamical fluctuation of triaxiality as is discussed in §2. In this respect the band in ^{182}Os[10] may be a possible candidate, for which the calculation in Fig.3 has been done.[5] Recently it is predicted[11] that the coupling of the wobbling mode to the quasiparticle orbits leads the signature-inversion of routhians in odd $\gamma < 0$ nuclei, which is observed in e.g. ^{167}Tm and is difficult to explain in the simple cranking model. This an example which indicates an importance of the wobbling motion in odd nuclei.

The characteristic staggering of $B(E2)_{\Delta I=\pm 1}$ as is shown in Fig.3 may be an important factor for the identification. This staggering is rather general and has been observed in the γ-vibrational bands at low-spin[1] though the staggering is not so pronounced because $Q_1^{(-)} \ll Q_2^{(-)}$ in this case (cf. eq.(1)). There is a definite phase rule between the RPA amplitudes of $Q_2^{(-)}$ and $Q_1^{(-)}$ for the triaxial deformation[5] and, for example, $B(E2)_{\Delta I=+1} >$ ($<$) $B(E2)_{\Delta I=-1}$ for $0 > \gamma > -60^\circ$ ($\gamma > 0$) so that the way of staggering depends on the sign of γ. Although such transitions are too weak and their measurements are not yet available, we hope that the new generation 4π crystal ball spectrometers make it possible in near future.

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