Proton–neutron interaction in rotating odd–odd nuclei and its effects on signature splitting

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Effects of the residual proton–neutron interaction in the rotating frame are studied by analyzing the signature inversion observed in odd–odd nuclei. In particular, the importance of configuration mixing induced by the interaction is stressed. Spectra given by the cranking model are improved appreciably but the effects of the residual interaction seem insufficient to reproduce experimental data.

Rotational bands built on high-j unique-parity states in odd-A nuclei decouple into two sequences. Energy levels with $I-j=even$ lie lower than those with $I-j=odd$; the former is called the favored (f) sequence and the latter is called the unfavored (u) sequence. The energy difference between these two sequences seen from a rotating frame is called the signature splitting because each sequence has a definite signature quantum number $r=\exp(-i\pi\alpha)$, where $I=\alpha$ mod 2. The magnitude of the signature splitting depends sensitively on the Fermi surface in high-j shells but its sign is never inverted in one-quasiparticle (1qp) bands. After the bandcrossings, i.e., in 3qp bands, say, $(\pi h_{11/2})^1(\nu i_{13/2})^2$, however, the sign is inverted systematically [1]. This phenomenon is called the signature inversion. It has been understood as a result of the positive gamma deformation driven by alignments of low-Ω two quasiparticles [2–5]. But this is insufficient since the signature inversion has also been observed in some nuclei where the alignments are believed to drive negative gamma deformations [1]. The coupling between the odd quasiparticle and gamma vibration is a promising candidate for another mechanism of the inversion. Note that the structure change of the vibrational excitation caused by the bandcrossing should be taken into account properly since the coupling makes the signature splitting larger in the case of 1qp bands with negative gamma deformations [6].

The signature inversion in odd-A nuclei seems to be understandable as a problem of a quasiparticle moving in triaxially deformed, vibrating and rotating potentials. In the case of odd–odd nuclei we consider un-like 2qp states, for instance, mid-$\Omega(\pi h_{11/2})^1 \otimes$ low-$\Omega(\nu i_{13/2})^1$. Since both the last quasiproton and the last quasineutron have two signature states, two configurations exist for each spin value based on an intrinsic 2qp state in general: $|\pi_{+i}\nu_{-i}\rangle$ and $|\pi_{-i}\nu_{+i}\rangle$ for even spins, $|\pi_{+i}\nu_{+i}\rangle$ and $|\pi_{-i}\nu_{-i}\rangle$ for odd spins. But, when one odd quasiparticle (τ) is decoupled whereas the other (τ') is relatively strongly-coupled, we can regard $|\tau_f'\tau_f\rangle$ and $|\tau_u'\tau_f\rangle$ as signature partners. Good examples can be found in the $N\simeq 90$ region where the $i_{13/2}$ neutron signature splitting is very large while that of the $h_{11/2}$ proton is small. The signature inversion in such systems, defined by $e'_{\pi_u \nu_f} - e'_{\pi_f \nu_f} < 0$, was observed earlier than that in odd-A nuclei, and has been described as a consequence of the positive gamma deformation driven by the low-Ω last quasineutron [7]. From this standpoint, its role is just to polarize the nuclear shape and then the signature splitting of the odd–odd nucleus reduces to $e'_{\pi_u} - e'_{\pi_f}$, the signature splitting of the last quasiproton moving in the polarized potential. In this sense the low-Ω quasineutron is a "spectator" while the mid-Ω quasiproton is a "participant" which corresponds to the last quasiparticle in odd-A nuclei. This picture is useful if the residual interaction between

the unpaired quasiparticles is sufficiently weak. But, generally speaking, the interaction between them and vibrations excited on the triaxially-deformed core nucleus should be taken into account. In the following, we investigate to what extent the cranking picture of the odd–odd nucleus is modified by the residual proton–neutron interaction by analyzing the signature inversion. As for the form of the interaction, we choose a Q–Q type one for the following reasons: (i) the gamma degree of freedom is believed to be the most important cause of signature dependent phenomena in odd-A nuclei; (ii) the force strength can be determined theoretically by requiring the shape selfconsistency between the potential and the density; (iii) we plan to perform quasiparticle–quasiparticle–vibration coupling calculations with phonons given by the RPA using the Q–Q interaction.

The outline of the numerical calculations is as follows: 2qp states were first constructed by diagonalizing cranked axial or triaxial Nilsson-plus-BCS potentials. The frequency-dependent shape evolution was neglected for simplicity. The model space was set so as to include a high-j shell in both proton and neutron parts, respectively; actually we adopted 10 (with $N_\pi = 5$) × 10 (with $N_\nu = 6$) [#1] × 2({(π_{+i}, ν_{-i}) and (π_{-i}, ν_{+i})} or {(π_{+i}, ν_{+i}) and (π_{-i}, ν_{-i})}) = 200 dimensional space for each signature sector ($r = +1$ or -1). The ordinary (nonstretched) Q–Q type proton–neutron interaction was diagonalized in this space. The interaction strength was taken to be twice as large as the so-called selfconsistent value in order to take account of the effects of the $\Delta N = 2$ couplings. In the following figures the cranking results and the results with the interaction matrix elements are shown.

Fig. 1 shows the theoretical signature splitting of the yrast $(\pi h_{11/2})^1 (\nu i_{13/2})^1$ band of ^{154}Tb calculated using an axially symmetric potential. This nucleus is typical of the $N = 89$ isotones which show strong signature inversion systematically. The cranking calculation (the curve denoted by "cr") gives positive, i.e., normal signature splitting because of axial symmetry. The diagonal Q_π–Q_ν interaction decreases the splitting by 100–200 keV and consequently the experimental data [8] are reproduced well ("diag"). Diagonalizing the off-diagonal interaction, however,
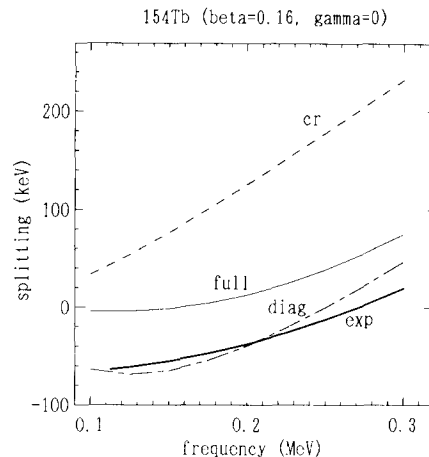

[#1] $N_\nu = 5$ for the Cs case discussed later.

Fig. 1. Calculated and experimental signature splittings of the yrast $(\pi h_{11/2})^1 (\nu i_{13/2})^1$ band of ^{154}Tb as a function of the rotational frequency. "Cr", "diag" and "full" denote the results of the cranking calculation, the calculation including the diagonal Q_π–Q_ν interaction matrix elements only, and the one including both the diagonal and the off-diagonal elements, respectively. The parameters used are $\beta^{(pot)} = 0.16$, $\gamma^{(pot)} = 0$ and $\Delta_\pi = \Delta_\nu = 1.0$ MeV. The data are taken from ref. [8].

the splitting increases again ("full"), namely, it weakens the inversion brought about by the diagonal interaction although the "full" curve is still better than the "cr" curve.

These results can be understood as follows: The diagonal elements produce signature-dependent energy shifts. The situation is similar to the first-order effect of the static triaxial deformation on the splitting in odd-A nuclei [5], and therefore, the occurrence of the inversion in such a nucleus with low Ω_ν is understandable. Since the difference between Ω_π and Ω_ν is big, the interaction acts repulsively in each signature sector as widely recognized [9]. The variation of the splitting, that is, the difference between the energy shifts, is proportional to the diagonal matrix element of $Q_2^{(+)} = (Q_{+2} + Q_{-2})/\sqrt{2}$ with respect to the "spectator" quasineutron state with the favored signature (see the top left part of fig. 1 in ref. [10] for a $\pi h_{11/2}$ case), whereas it is proportional to the expectation value $\langle Q_2^{(+)} \rangle$ in the static deformation case.

Diagonalization of the off-diagonal interaction matrix elements causes configuration mixing among $\pi^1 \nu^1$ states with the same parity and signature. The probabilities of the two largest components out of 200 for each signature sector are shown in fig. 2. We can
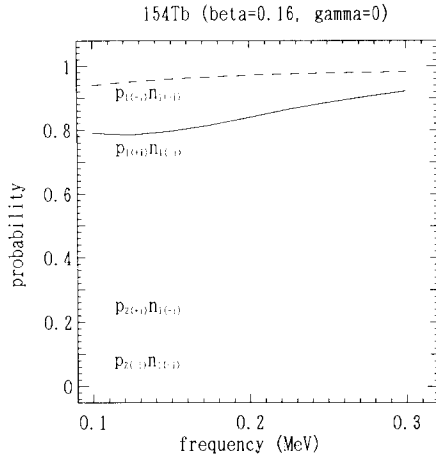
Fig. 2. The probabilities of the major 2qp components among 200 in the wave function of the lowest-energy eigenstate in each sector calculated as a function of the rotational frequency. The parameters used are the same as in fig. 1. The $r= +1$ (favored) and -1 (unfavored) sectors are shown by solid and broken lines, respectively. The subscripts are the ordinal numbers of single-quasiparticle states counted from the lowest energy.

Fig. 3. A schematic drawing of the single-particle wave functions in the space of Ω, the z component of the single-particle angular momentum.

see from this figure that the lowest-energy state in the unfavored signature ($r= -1$) sector is an almost pure 2qp state but that in the favored signature ($r= +1$) sector contains the 2nd unperturbed 2qp state appreciably. The mixing of the signature-conjugate component in each sector, $\pi_{1(-i)}\nu_{1(+i)}$ for $r= +1$ and $\pi_{1(+i)}\nu_{1(+i)}$ for $r= -1$, is negligible. As expected generally, these eigenstates approach to pure 2qp states as the frequency increases. The configuration mixing discussed above lowers the lowest-energy states in both sectors irrespective of their Fermi surfaces. The magnitude of the energy variation is larger in the favored sector, and accordingly the signature splitting is driven to the normal sign by the off-diagonal interaction matrix elements. The reason is as follows: Favored states contain more $\Omega= \frac{1}{2}$ components than unfavored states because the wave functions of the latter have one more node in the space of Ω than their signature partners as depicted schematically in fig. 3. Therefore the $\pi_f\nu_f$ states have larger interaction matrix elements with each other than the $\pi_u\nu_f$ states due to the ν_f with large $\Omega= \frac{1}{2}$ components. Summing up the effects of the diagonal and the off-diagonal interaction matrix elements, the lowest favored state is pushed up more strongly than the lowest unfavored state. A similar tendency was reported
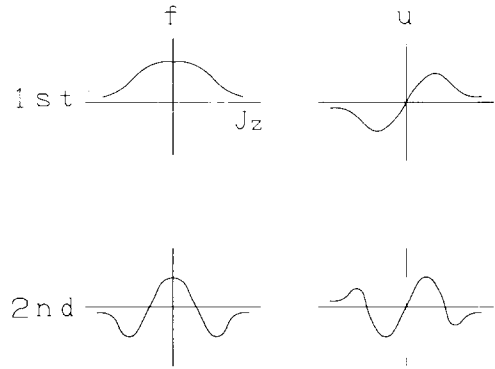
in ref. [11] where $\gamma= -40°$ was assumed for Tl isotopes.

Now we return to the relation between the data and the calculation. The example presented above indicates that the effect of the Q_π-Q_ν interaction is appreciable but insufficient to reproduce the observed signature inversion. This result is qualitatively consistent with a particle–rotor model calculation for an odd-A nucleus, [157]Ho, done by Hamamoto [12]. She needed an order of magnitude strong interaction to reproduce the data. A natural extension of the present model is to incorporate the positive gamma deformation at the same time as the residual interaction. Actually large signature inversion can be produced already in the cranking calculation if β and γ are treated as free parameters. But, when the information on electromagnetic transition rates is available, the magnitude of Q_t ($\alpha\sqrt{B(E2:\Delta I= 2)}$) and the signature dependence of $B(M1)$ restrict the ranges of β and γ, respectively. Although such information is not sufficient in the $A \simeq 160$ region where the signature inversion has been discussed, there exist such data in other mass region.

Figs. 4 and 5 graph the results for [124]Cs in which both the proton and neutron Fermi surfaces lie in the $h_{11/2}$ shell. Here the potential parameters were determined so as to reproduce the preliminary Q_t data [14] (3.9 ± 0.5 e b for $E_\gamma= 869$ keV and 3.5 ± 1.5 e b for $E_\gamma= 973$ keV) approximately and to be consistent with the systematics around this nucleus [15]. The calculated Q_t is about 3.6 e b and this value is almost
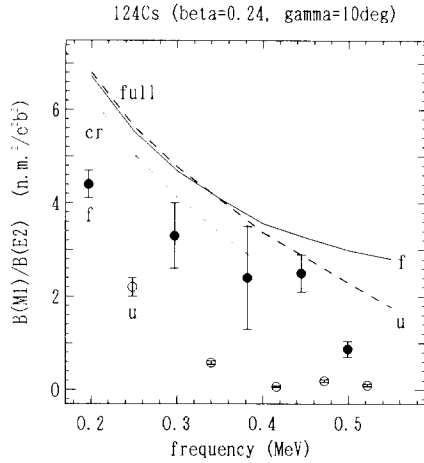
Fig. 4. Calculated and experimental $B(M1:\Delta I=1)/B(E2:\Delta I=2)$ ratios of the yrast $(\pi h_{11/2})^1(\nu h_{11/2})^1$ band in ^{124}Cs as a function of the rotational frequency. The thin and thick lines represent the results which correspond to the "cr" and the "full" calculations but the effects of the interaction was taken into account only in the numerator in the latter [see the text]. In both cases, the solid and broken lines represent the transitions from the favored sequence and those form the unfavored sequence, respectively. The corresponding data are shown by filled and open circles, respectively. The parameters used are $\beta^{(\mathrm{pot})}=0.24$, $\gamma^{(\mathrm{pot})}=10°$ and $\Delta_\pi=\Delta_\nu=1.0$ MeV. The data are taken form ref. [3].
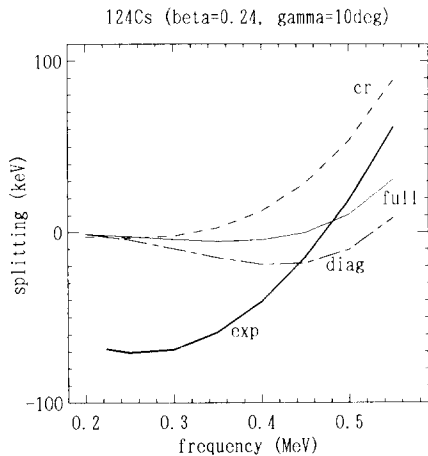

Fig. 5. Calculated and experimental signature splittings of the yrast $(\pi h_{11/2})^1(\nu h_{11/2})^1$ band of ^{124}Cs as a function of the rotational frequency. The notations are the same as in fig. 1. The parameters used are the same as in fig. 4. The data are taken from ref. [13].

independent of the frequency. The experimental and theoretical $B(M1:\Delta I=1)/B(E2:\Delta I=2)$ ratios are presented in fig. 4. The observed signature dependence [2], $B(M1:f\to u)>B(M1:u\to f)$, and the overall frequency dependence are reproduced semi-quantitatively. The configuration mixing due to the off-diagonal interaction matrix elements weakens the signature dependence slightly; this is consistent with the effect on the signature splitting in energy discussed later. As for the signature-average magnitude, however, the calculation gives larger values than the data. This deviation originates from the numerator, $B(M1)$. This is a common feature in cranking calculations which use the calculated g-factor [17,18],

$$g_{\mathrm{ref}}=\frac{\langle\mu_x\rangle_{\mathrm{ref}}}{\langle J_x\rangle_{\mathrm{ref}}},$$

with reference states consisting of the even–even core and aligned quasiparticle(s), in

$$\mu_{-1}=(g_l-g_{\mathrm{ref}})l_{-1}+(g_s^{(\mathrm{eff})}-g_{\mathrm{ref}})s_{-1}.$$

The reference state consists of the even–even core and the lowest energy $(\pi h_{11/2})^1$ in the present odd–odd case [3], while it includes the core and aligned two quasiparticles in the case of 3qp bands in odd-A nuclei (see figs. 5–8 in ref. [6], for example). The magnitudes of the $B(M1)$ are made slightly larger by the interaction. Note that effects of the interaction on the denominator were neglected in this calculation because the $B(E2:\Delta I=2)$ is determined predominantly by the collective property while the interaction changes the 2qp part only.

Fig. 5 shows the signature splittings calculated using the parameters which satisfy approximately the restrictions from the transition rates, in comparison to the data. The residual $Q_\pi-Q_\nu$ interaction produces weak signature inversion but the observed splitting cannot be reproduced especially at the low spin region within the present framework. The relation between the signature splitting in quasiparticle energy and the signature dependence of $B(M1)$ in axially symmetric nuclei is known widely [19]. And small deviations between the inversion frequency regions of the energy and $B(M1)$ can be explained as a kind

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[2]　The spin assignments conform to ref. [16].

[3]　This is the dominant component even after the introduction of the residual interaction.

of modification due to the positive gamma deformation [5]. But, in the present case, both the inversion in energy and the normal signature dependence of $B(M1)$ are strong. This fact makes it difficult to reproduce the data within the present framework.

Some preliminary calculations seem to indicate that the bigger the Ω of the "participant" nucleon becomes, the more conspicuous the deviation between the data and the calculations within the cranking plus $Q_\pi-Q_\nu$ interaction becomes at the low spin region. We encountered a similar situation in odd-A nuclei [20]: The static triaxial deformation causes signature-dependent staggering as a function of spin in the $B(E2:\Delta I=1)$. The phase of the staggering is determined uniquely by the sign of γ in the cranking calculation irrespective of the position of the Fermi surface. In a particle–rotor model calculation in which the angular momentum coupling is treated appropriately and hence better results are expected for low spins, however, the cranking phase rule applies only to low-Ω nuclei (see fig. 8 of ref. [21]). This contradiction was recovered by introducing the couplings between the last odd quasiparticle and gamma vibration into the cranking calculation. The exchange interaction in the interacting boson–fermion model shows a similar effect [22] but unusually strong one is necessary in order to alter the signature dependence of the $B(E2:\Delta I=1)$ [23]. This can be understood by considering such a property that the odd-spin gamma vibrational excitation changes its character gradually to wobbling motion as spin and triaxial deformation increase [24,25]. In other words, the shape fluctuation undergoes a change to a fluctuation of the rotational axis seen from the principal axis frame. The coupling with this mode, therefore, introduces nonuniform rotation of the triaxially deformed nucleus on the quasiparticle motion within a small amplitude approximation. Learning from this experience, the gamma vibrational effects will also be important in odd–odd nuclei.

In summary, we have studied the spectra of rotating odd–odd nuclei aiming to see to what extent the cranking picture is modified by the residual $Q_\pi-Q_\nu$ interaction. The signature splittings in odd–odd nuclei given by the cranking model are improved appreciably by the residual interaction but its effect is insufficient to reproduce the data especially at the low spin region. The calculations taking account of the gamma vibrational effects will be reported separately.

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