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COHERENT EFFECTS OF STATIC AND DYNAMIC TRIAXIALITIES ON $B(E2:\Delta I=1)$

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ABSTRACT

The signature dependence of $B(E2:I\to I-1)$ in rotating odd-A nuclei brought about by static and dynamic triaxial deformations is studied by means of the quasiparticle-vibration coupling approach based on the RPA in a rotating frame. After presenting two kinds of phase rules of the staggering as a function of spin, their competition is discussed.

1. INTRODUCTION

Excited quasiparticles as well as collective rotation polarize the nuclear shape. This polarization effect has mainly been discussed in terms of static deformations: β, γ, ϵ_4 and so on. Although the existence of beta vibration in axially deformed nuclei is recognized, gamma vibration and static triaxial deformation have often been treated as conflicting concepts. But, needless to say, they should be taken into account simultaneously from the theoretical point of view. After both were considered, we can make a judgment whether one of them is negligible or both are important.

We proposed in ref.1) such a framework that both can be taken into consideration at the same time and have been studying various quantities in rotating odd-A nuclei: energy spectra, $B(M1)$ and $B(E2)$ values and mixing ratios. As for energy spectra and $B(M1:I\to I-1)/B(E2:I\to I-2)$ values, rich information is available. In contrast, data of $B(E2:I\to I-1)$ are limited although they are expected to carry more direct knowledge of triaxiality.

In sect.2, we discuss the signature dependence of $B(E2: I \to I-1)$ stemming from the static triaxial deformation of rotating potentials and the relation to the particle-rotor model result. In sect.3, the effect of the gamma-vibrational contribution is discussed paying attention to its shell-filling dependence. The competition between the effects of static and dynamic triaxialities is studied numerically in sect.4. Summary is given in sect.5.

2. SIGNATURE DEPENDENCE OF $E2$ MATRIX ELEMENTS DUE TO STATIC TRIAXIAL DEFORMATION

The effective principal-axis (PA) frame operator for $E2(\Delta I = 1)$ transitions consists of three parts, the rotational part, the vibrational part and the odd-quasiparticle part:

$$Q'_{-1}{}^{(\mathrm{PA})} = Q'_{-1}{}^{(\mathrm{rot})} + Q'_{-1}{}^{(\mathrm{vib})} + Q'_{-1}{}^{(\mathrm{qp})} \quad . \tag{1}$$

The contribution of the last term is much smaller than that of others. The concrete form of the rotational part is

$$Q'_{-1}{}^{(\mathrm{rot})} = \frac{i}{\sqrt{2}}\{-\sqrt{3} < Q_0^{(+)} > \frac{J_z^{(\mathrm{qp})}}{I_0} + < Q_2^{(+)} > \left(2\frac{iJ_y^{(\mathrm{qp})}}{I_0} + \frac{J_z^{(\mathrm{qp})}}{I_0}\right)\} \quad , \tag{2}$$

where Q'_{-1} is quantized along the x (cranking) axis while $< Q_K^{(+)} >$ $(K = 0,2)$ along the z axis, and I_0 denotes the angular momentum of the even-even core. In the cranking model, a triaxially deformed core (potential) rotates around a principal axis and then its main effect on $E2(\Delta I = 1)$ matrix elements appears via $< Q_2^{(+)} >$ in eq.(2). An identity derived from the commutator between the cranking Hamiltonian

$$h' = h_{\mathrm{sph}} - \sum_{K=0,2} \alpha_K Q_K^{(+)} - \hbar\omega_{\mathrm{rot}} J_x \tag{3}$$

and J_z,

$$-\Delta e' \langle \mathrm{f}|\, J_z \,|\mathrm{u}\rangle = \hbar\omega_{\mathrm{rot}} \langle \mathrm{f}|\, iJ_y \,|\mathrm{u}\rangle + 2\alpha_2 \langle \mathrm{f}|\, Q_2^{(-)} \,|\mathrm{u}\rangle \tag{4}$$

with the signature splitting $\Delta e' = e'_{\mathrm{u}} - e'_{\mathrm{f}}$, assures a definite phase relation between the single-quasiparticle matrix elements of J_z and iJ_y in eq.(2) when the triaxial parameter $\alpha_2 \propto \sin\gamma^{(\mathrm{pot})}$ is small. Consequently the signature dependence due to static triaxial deformation is determined by the sign of $< Q_2^{(+)} >$, i.e., of γ as

$$B(E2: \mathrm{f} \to \mathrm{u}) \gtrless B(E2: \mathrm{u} \to \mathrm{f}) \quad \text{for} \quad \gamma \gtrless 0 \quad , \tag{5}$$

where f and u stand for the favored and the unfavored states, respectively.

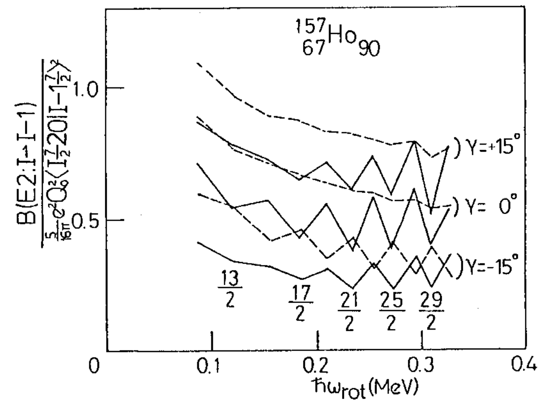

Fig.1. The calculated $B(E2: I \to I-1)$ divided by the rotational values. The solid and broken lines represent the results with and without taking into account the gamma-vibrational contributions, respectively. Parameters used are $\beta^{(\mathrm{pot})} = 0.20$, $\Delta_\mathrm{p} = 1.21\mathrm{MeV}$ and $\Delta_\mathrm{n} = 1.25\mathrm{MeV}$. (From ref.3).)

In contrast, static triaxial deformation in the particle-rotor model is accompanied by the fluctuation of the rotational axis. This is the reason why the signature dependence in a negative-gamma case in Hamamoto's particle-rotor calculation is inverted relative to the cranking prediction, eq.(5), when Ω/I is not small (see fig.8 of ref.2)). Here I and Ω denote the total spin and the angular momentum projection onto the z axis, respectively. At least a part of this effect of the fluctuation can be taken into account by the coupling with gamma-vibrational phonons as shown in fig.1; $B(E2 : \mathrm{f} \rightarrow \mathrm{u})$s are enhanced relative to $B(E2 : \mathrm{u} \rightarrow \mathrm{f})$s both in $\gamma = +15°$ and $-15°$ cases as in the particle-rotor model result. This is supported by such a scenario that the gamma vibration in rotating triaxial nuclei changes its character gradually to wobbling motion as $\omega_{\rm rot}$ increases[4)-6)].

3. SIGNATURE DEPENDENCE OF $E2$ MATRIX ELEMENTS DUE TO DYNAMIC TRIAXIAL DEFORMATION

The vibrational part in eq.(1) is given by

$$Q'_{-1}{}^{(\mathrm{vib})} = \sum_n \{[X_{n(-)}, Q'_{-1}]_{\mathrm{RPA}} X^\dagger_{n(-)} - \left[X^\dagger_{n(-)}, Q'_{-1}\right]_{\mathrm{RPA}} X_{n(-)}\} \quad . \qquad (6)$$

Only the gamma-vibrational phonon in the negative-signature ($r = exp(-i\pi\alpha) = -1$) sector, $X^\dagger_{\gamma(-)}$, is taken into account in the following. With the aid of the relation,

$$Q'_{-1} = \frac{i}{\sqrt{2}}\left(Q_1^{(-)} - Q_2^{(-)}\right) \quad , \qquad (7)$$

the matrix element can be written, in the first order with respect to the quasiparticle-vibration coupling (QVC), as

$$\left.\begin{array}{l}(\mathrm{f}\rightarrow\mathrm{u})\\(\mathrm{u}\rightarrow\mathrm{f})\end{array}\right\} \simeq \frac{i}{\sqrt{2}}\{-\sqrt{3} < Q_0^{(+)} > \frac{\langle \mathrm{f}|\, J_z \,|\mathrm{u}\rangle}{I_0} + \frac{2}{\hbar\omega_{\gamma(-)}}\left(\chi_1^{(-)}(T_1^{(-)})^2 \langle \mathrm{f}|\, Q_1^{(-)} \,|\mathrm{u}\rangle \pm \chi_2^{(-)}(T_2^{(-)})^2 \langle \mathrm{f}|\, Q_2^{(-)} \,|\mathrm{u}\rangle\right)\} \quad , \qquad (8)$$

where

$$T_K^{(-)} = \left[Q_K^{(-)}, X^\dagger_{\gamma(-)}\right]_{\mathrm{RPA}} \quad (K = 1, 2) \qquad (9)$$

is the transition amplitude associated with the gamma-vibrational phonon, and $\hbar\omega_{\gamma(-)}$ and $\chi_K^{(-)}$ ($K = 1, 2$) are the excitation energy of the phonon and the

strength of the Q-Q interaction, respectively. The equilibrium shape is assumed to be axially symmetric and the odd-quasiparticle term is neglected here for simplicity. It is obvious from eq.(8) that the signature dependence due to gamma vibration is determined by the relative sign between $\langle \mathrm{f}|\, J_z \,|\mathrm{u}\rangle$ in the main term and $\langle \mathrm{f}|\, Q_2^{(-)} \,|\mathrm{u}\rangle$ in the vibrational term.

The next task is to study this relative sign. The quadrupole operators with $r = -1$ in the single-j shell model are represented by replacing the coordinate $\vec{x}$ by the angular momentum $\vec{J}$ as follows:

$$\begin{aligned} Q_1^{(-)} &= -2\sqrt{3c_0}\frac{1}{2}\{J_x, J_z\} \quad , \\ Q_2^{(-)} &= 2\sqrt{3c_0}\frac{1}{2}\{J_x, iJ_y\} \quad , \end{aligned} \qquad (10)$$

with $\{,\}$ signifying an anticommutator, and

$$c_0 = \sqrt{\frac{5}{16\pi}}\frac{q_0}{j(j+1)} \quad , \qquad (11)$$

where q_0 is a constant with dimension $[L^2]$. We now assume that J_x in eq.(10) can be replaced by an aligned angular momentum when we deal with the lowest-energy quasiparticle states. Using eq.(4) with $\alpha_2 = 0$ we then obtain

$$\Delta e' \langle \mathrm{f}|\, Q_1^{(-)} \,|\mathrm{u}\rangle \simeq \hbar\omega_{\rm rot} \langle \mathrm{f}|\, Q_2^{(-)} \,|\mathrm{u}\rangle \quad . \qquad (12)$$

The relation (12) implies that $\langle \mathrm{f}|\, Q_2^{(-)} \,|\mathrm{u}\rangle$ and $\langle \mathrm{f}|\, Q_1^{(-)} \,|\mathrm{u}\rangle$ have the same sign, except when the signature inversion[7)] occurs, in high-j cases in which the single-j approximation holds well. Accordingly we can state that the signature dependence of $B(E2)$ is determined by the relative sign between $\langle \mathrm{f}|\, J_z \,|\mathrm{u}\rangle$ and $\langle \mathrm{f}|\, Q_1^{(-)} \,|\mathrm{u}\rangle$.

Assuming axial symmetry ($\alpha_2 = 0$), an identity

$$-\Delta e' \langle \mathrm{f}|\, iJ_y \,|\mathrm{u}\rangle = \hbar\omega_{\rm rot} \langle \mathrm{f}|\, J_z \,|\mathrm{u}\rangle + \sqrt{3\alpha_0} \langle \mathrm{f}|\, Q_1^{(-)} \,|\mathrm{u}\rangle \qquad (13)$$

is derived from the commutator between h' and iJ_y besides

$$-\Delta e' \langle \mathrm{f} | J_z | \mathrm{u} \rangle = \hbar\omega_{\mathrm{rot}} \langle \mathrm{f} | iJ_y | \mathrm{u} \rangle \quad . \tag{4'}$$

Using these identities,

$$\frac{\langle \mathrm{f} | Q_1^{(-)} | \mathrm{u} \rangle}{\langle \mathrm{f} | J_z | \mathrm{u} \rangle} = \frac{\hbar\omega_{\mathrm{rot}}}{\sqrt{3}\alpha_0} \{ \left(\frac{\Delta e'}{\hbar\omega_{\mathrm{rot}}} \right)^2 - 1 \} \tag{14}$$

is derived. Therefore we have obtained a phase rule of the signature dependence due to gamma vibration:

$$B(E2 : \mathrm{f} \to \mathrm{u}) \lesseqgtr B(E2 : \mathrm{u} \to \mathrm{f}) \quad \text{for} \quad \Delta e' \gtreqless \hbar\omega_{\mathrm{rot}} \quad . \tag{15}$$

Since $\Delta e'$ decreases as the chemical potential λ increases in a high-j shell, the ratio $B(E2 : \mathrm{f} \to \mathrm{u})/B(E2 : \mathrm{u} \to \mathrm{f})$ also varies with λ as shown in fig.2.

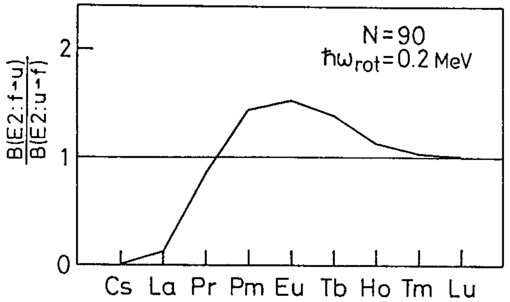

Fig.2. The calculated ratio of $B(E2 : I \to I-1)$ representing the signature dependence at $\gamma = 0$ given by the quasiparticle-vibration coupling (QVC) model as a function of Z. Parameters used are $\beta^{(\mathrm{pot})} = 0.20$ and $\Delta_{\mathrm{p}} = \Delta_{\mathrm{n}} = 1.0$MeV. (From ref.8).)

The result shown in fig.2 is consistent with that of all the previous works: i) When the Fermi surface lies at the mid $\pi h_{11/2}$ shell region, the signature splitting is smaller than the rotational frequency. In this case, the gamma-vibrational contributions enhance $B(E2 : \mathrm{f} \to \mathrm{u})$. This is the case discussed by Ikeda[9] and in our previous works[3),1)]. ii) When the Fermi surface lies low in the $\pi h_{11/2}$ shell, the signature splitting is larger than the rotational frequency. In this case, the gamma-vibrational contributions enhance $B(E2 : \mathrm{u} \to \mathrm{f})$. This is the case discussed by Onishi et al.[10)]

4. COMPETITION BETWEEN THE EFFECTS OF STATIC AND DYNAMIC TRIAXIAL DEFORMATIONS

The phase rules of the signature dependent staggering of $B(E2 : I \to I-1)$ in rotating odd-A nuclei as a function of spin due to static and dynamic triaxialities have been discussed analytically and numerically in sects.2 and 3, respectively. The next step is to study their competition in coexistent systems.

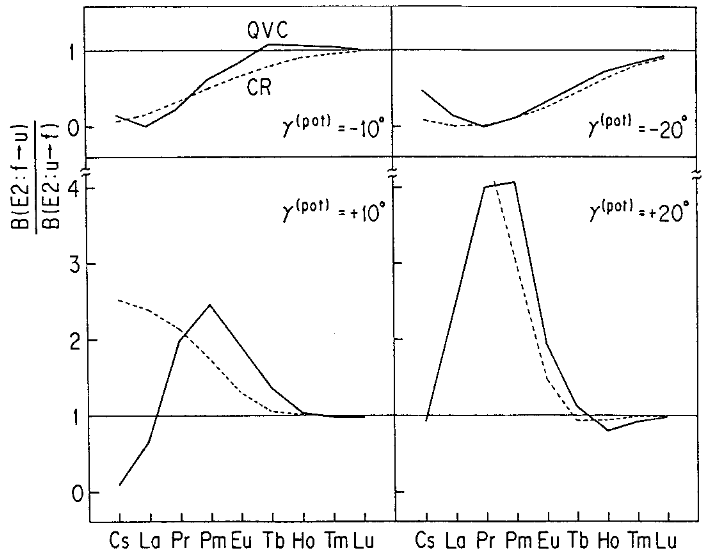

Fig.3. Same as fig.2 but for $\gamma \neq 0$. The cranking (CR) values are also shown by the broken lines. (From ref.11).)

Numerical calculation was performed for $\gamma^{(\mathrm{pot})} = \pm 10°$ and $\pm 20°$ under the same condition as the $\gamma^{(\mathrm{pot})} = 0$ case in sect.3. The result is presented in fig.3. The broken lines in the figure indicate the results without the vibrational term (see eq.(1)). The signature dependence of $B(E2)$, i.e., the deviation of the ratio from unity, stems predominantly from the term proportional to $< Q_2^{(+)} > \cdot J_y$ in eq.(2). The phase rule (5), derived assuming small γ, holds well except for the positive-gamma cases of high-λ nuclei in which the signature inversion takes place.

The solid lines include the gamma-vibrational effects superimposed on the cranking results. We here note that the phonons were constructed within the RPA based on each triaxially deformed rotating mean field. For $|\gamma^{(\mathrm{pot})}| = 10°$, the phase rule due to gamma vibration:

$$\exists\lambda'_{\mathrm{crit}}:\ B(E2:\mathrm{f}\to\mathrm{u}) \gtrless B(E2:\mathrm{u}\to\mathrm{f}) \quad \text{for} \quad \lambda \gtrless \lambda_{\mathrm{crit}} \qquad (15')$$

derived using the first-order perturbation holds although λ_{crit} varies depending on γ. In contrast, the ratio is always smaller than unity for $\gamma^{(\mathrm{pot})} = -20°$ while it is larger than unity for $\gamma^{(\mathrm{pot})} = +20°$ except at the signature inversion region and at Cs. In other words, the effect of static gamma deformation is stronger than that of gamma vibration at $|\gamma^{(\mathrm{pot})}| = 20°$. The zero-point amplitude γ_0 associated with gamma vibration calculated at $\hbar\omega_{\mathrm{rot}} = 0$ and $\gamma^{(\mathrm{pot})} = 0$ is about 15° in these nuclei. According to it, we can conclude that the phase rule (15) holds for the nuclei situated at the vibrational region, $|\gamma^{(\mathrm{pot})}| < \gamma_0$, whereas the signature dependence is determined by the sign of $\gamma^{(\mathrm{pot})}$ when $|\gamma^{(\mathrm{pot})}|$ is larger than γ_0.

Static triaxial deformation γ has been treated as an input parameter up to now. But it depends on the shell-filling in general. Two kinds of standpoints are possible for how to determine γ in the QVC model. The first is such that the polarization effect of the odd-quasiparticle can be taken into account by the equilibrium shape of the even-even core and the coupling between the quasiparticle moving in the potential with this shape and the vibration of the core around this shape. This accords with the nuclear-field-theoretical approach. Then, negative-gamma deformation is appropriate for one-quasiparticle bands because of the collective rotation of the core while positive-gamma deformation is appropriate for the three-quasiparticle bands in the $N = 90$ isotones because the equilibrium γ of the s-bands of the even-even nuclei in this region is positive.

The second standpoint is such that the QVC calculation should be done based on the equilibrium shape of the odd-mass system. Then, since the equilibrium γ of the odd-mass system varies as

$$\exists\lambda'_{\mathrm{crit}}:\ \gamma \lesseqgtr 0 \quad \text{for} \quad \lambda \gtrless \lambda'_{\mathrm{crit}} \ , \qquad (16)$$

the cranking value will show

$$\exists\lambda'_{\mathrm{crit}}:\ B(E2:\mathrm{f}\to\mathrm{u}) \lesseqgtr B(E2:\mathrm{u}\to\mathrm{f}) \quad \text{for} \quad \lambda \gtrless \lambda'_{\mathrm{crit}} \qquad (17)$$

in consequence of the phase rule (5). The QVC contribution which shows the phase rule (15′) is superimposed on the cranking value. The cranking and QVC effects, therefore, contribute in the opposite direction of each other at both ends of high-j shells while they can be in phase at mid shell region depending on λ_{crit} and λ'_{crit}. At the present stage, it is an open question which standpoint is appropriate. This should be studied from the many-body theoretical view point.

5. SUMMARY

We have studied the signature dependence of $B(E2: I \to I-1)$ in rotating odd-A nuclei due to static and dynamic triaxialities. In sect.2, the phase rule of the staggering as a function of spin stemming from the static deformation has been presented; this rule holds not only in the cranking model but also in the particle-rotor model when Ω/I is small. When it is not small, the fluctuation of the rotational axis in the particle-rotor model inverts the phase of the staggering; this effect can be incorporated into the cranking description by considering the quasiparticle-vibration coupling. The gamma-vibrational effect in axially symmetric nuclei has been discussed in sect.3. In particular, its shell-filling dependence has been clarified analytically. In sect.4, the competition between the effects of static and dynamic triaxialities has been studied numerically, and it has been shown that the phase rule due to gamma vibration survives when $|\gamma^{(pot)}|$ is smaller than the zero-point amplitude γ_0. The principle how to choose the appropriate γ in the quasiparticle-vibration coupling approach has also been mentioned.

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