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ROTATIONAL CHARACTER CHANGE FROM GAMMA VIBRATION TO WOBBLING MOTION

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Abstract: The character change of the gamma-vibrational excitation in rotating triaxial nuclei to the wobbling motion is studied paying attention to the rotational K-mixing in its wave function. Based on an analytic expression for the ratio of the transition amplitudes, a kind of relation between the microscopic and macroscopic descriptions of the nuclear wobbling motion and the static-γ dependence of the interband $B(E2:\Delta I=1)$ values are discussed by means of the RPA.

The Coriolis and centrifugal forces bring about changes in the structure of rotating nuclei. This structure change has two aspects. One is gradual change within each rotational band which appears as the rotational K-mixing in its wave function. The other is abrupt change associated with quasiparticle alignments. An interesting example of the former is the character change of the gamma vibration with negative signature ($r=\exp(-i\pi\alpha)=-1$) to the wobbling motion in rotating triaxial nuclei. This was predicted first by Mikhailov and Janssen within the framework of the RPA[1]. Marshalek generalized their discussion and clarified a kind of relation[2] to the macroscopic model of Bohr and Mottelson[3].

We study in this paper the K-mixing in vibrational wave functions analytically by evaluating the transition amplitudes to the yrast band and present another kind of relation between the microscopic and macroscopic descriptions of the wobbling motion. The static-γ dependence of the rotational behavior of the interband $B(E2:\Delta I=1)$ values is also discussed.

First of all, we review briefly the macroscopic wobbling model presented by Bohr and Mottelson. The creation operator of the wobbling mode is defined, aside from an overall phase, as

$$X_{\mathrm{W}}^{\dagger}=\frac{1}{\sqrt{2<J_x>}}\{(x-y)iJ_y^{(\mathrm{PA})}-(x+y)J_z^{(\mathrm{PA})}\} \quad , \tag{1}$$

where J_i's with a superscript (PA) denote the principal-axis frame components. The amplitudes x and y are given by

$$\left.\begin{matrix} x^2 \\ y^2 \end{matrix}\right\} = \frac{1}{2}\left(\frac{\alpha}{\sqrt{\alpha^2-\beta^2}}\pm 1\right) \quad , \tag{2}$$

with

$$\begin{aligned} \frac{\alpha}{\hbar^2} &= \frac{\hbar\omega_{\mathrm{rot}}}{2}\left(\frac{\mathcal{J}_x}{\mathcal{J}_y}+\frac{\mathcal{J}_x}{\mathcal{J}_z}-2\right) \quad , \\ \frac{\beta}{\hbar^2} &= \frac{\hbar\omega_{\mathrm{rot}}}{2}\left(\frac{\mathcal{J}_x}{\mathcal{J}_y}-\frac{\mathcal{J}_x}{\mathcal{J}_z}\right) \quad , \end{aligned} \tag{3}$$

in terms of the rotational frequency and moments of inertia ($\mathcal{J}_x > \mathcal{J}_y, \mathcal{J}_z$ and $\mathcal{J}_y \neq \mathcal{J}_z$). The excitation energy is given as another function of them:

$$\hbar\omega = \hbar\omega_{\mathrm{rot}}\sqrt{\frac{(\mathcal{J}_x-\mathcal{J}_y)(\mathcal{J}_x-\mathcal{J}_z)}{\mathcal{J}_y\mathcal{J}_z}} \quad , \tag{4}$$

like that in the classical mechanics. The interband electric-quadrupole transition rate between the wobbling and yrast states is given by

$$B(E2:I_{\mathrm{W}}-(I-1)_{\mathrm{yr}})=\frac{1}{<J_x>}(\sqrt{3}<Q_0'>x-\sqrt{2}<Q_2'>y)^2 \quad , \tag{5}$$

where Q_0' and Q_2' are quantized along the x axis.

Next, we turn to the RPA description of the wobbling motion. The coupled RPA dispersion equation for the pairing plus quadrupole interaction decouples to two sectors according to the signature quantum number[4]. The negative-signature sector includes $Q_K^{(-)}$ $(K=1,2)$, which are defined as

$$Q_K^{(\pm)}=\frac{1}{\sqrt{2(1+\delta_{K0})}}(Q_{+K}\pm Q_{-K}) \quad . \tag{6}$$

Therefore the excitation energy ω of eigen modes with $r=-1$.

$$X_n^{(-)\dagger}=\sum_{\mu\nu}{}''\{\psi_n(\mu\nu)a_\mu^\dagger a_\nu^\dagger+\phi_n(\mu\nu)a_\nu a_\mu\} \quad , \tag{7}$$

is determined by a two-dimensional dispersion determinant as

$$\begin{vmatrix} \sum''_{\mu\nu} \frac{2E_{\mu\nu}(Q_1^{(-)}(\mu\nu))^2}{E_{\mu\nu}^2-(\hbar\omega)^2} - \frac{1}{\kappa} & \sum''_{\mu\nu} \frac{2\hbar\omega Q_1^{(-)}(\mu\nu)Q_2^{(-)}(\mu\nu)}{E_{\mu\nu}^2-(\hbar\omega)^2} \\ \sum''_{\mu\nu} \frac{2\hbar\omega Q_2^{(-)}(\mu\nu)Q_1^{(-)}(\mu\nu)}{E_{\mu\nu}^2-(\hbar\omega)^2} & \sum''_{\mu\nu} \frac{2E_{\mu\nu}(Q_2^{(-)}(\mu\nu))^2}{E_{\mu\nu}^2-(\hbar\omega)^2} - \frac{1}{\kappa} \end{vmatrix} = 0 \quad . \tag{8}$$

Mikhailov and Janssen showed that, when $< Q_2^{(+)} > \neq 0$, eq.(8) could be cast into the form[1]:

$$(\omega^2 - \omega_{\rm rot}^2) \begin{vmatrix} A(\omega) & C(\omega) \\ B(\omega) & D(\omega) \end{vmatrix} = 0 \quad , \tag{9}$$

where

$$\begin{aligned} A(\omega) &= \omega \mathcal{J}_y(\omega) - \omega_{\rm rot}\mathcal{J}_{yz}(\omega) \quad , \\ B(\omega) &= \omega_{\rm rot}(\mathcal{J}_y(\omega) - \mathcal{J}_x) - \omega \mathcal{J}_{yz}(\omega) \quad , \\ C(\omega) &= \omega_{\rm rot}(\mathcal{J}_z(\omega) - \mathcal{J}_x) - \omega \mathcal{J}_{yz}(\omega) \quad , \\ D(\omega) &= \omega \mathcal{J}_z(\omega) - \omega_{\rm rot}\mathcal{J}_{yz}(\omega) \quad , \end{aligned} \tag{10}$$

with

$$\begin{aligned} \mathcal{J}_x &= \frac{< J_x >}{\hbar\omega_{\rm rot}} \quad , \\ \mathcal{J}_y(\omega) &= \sum_{\mu\nu}{}'' \frac{2E_{\mu\nu}(iJ_y(\mu\nu))^2}{E_{\mu\nu}^2 - (\hbar\omega)^2} \quad , \\ \mathcal{J}_z(\omega) &= \sum_{\mu\nu}{}'' \frac{2E_{\mu\nu}(J_z(\mu\nu))^2}{E_{\mu\nu}^2 - (\hbar\omega)^2} \quad , \\ \mathcal{J}_{yz}(\omega) &= \sum_{\mu\nu}{}'' \frac{2\hbar\omega i J_y(\mu\nu)J_z(\mu\nu)}{E_{\mu\nu}^2 - (\hbar\omega)^2} \quad . \end{aligned} \tag{11}$$

Here the angular-momentum components refer to the uniformly-rotating (UR) frame. The first factor in eq.(9) gives the Nambu-Goldstone mode while the second gives collective and non-collective normal modes. Marshalek obtained another expression equivalent to $A(\omega)D(\omega) - B(\omega)C(\omega) = 0$ [ref.2)]:

$$(\hbar\omega)^2 = (\hbar\omega_{\rm rot})^2 \frac{(\mathcal{J}_x - \mathcal{J}_y^{\rm (eff)}(\omega))(\mathcal{J}_x - \mathcal{J}_z^{\rm (eff)}(\omega))}{\mathcal{J}_y^{\rm (eff)}(\omega)\mathcal{J}_z^{\rm (eff)}(\omega)} \quad , \tag{12}$$

where

$$\begin{aligned} \mathcal{J}_y^{\rm (eff)}(\omega) &= \mathcal{J}_y(\omega) + \frac{B(\omega)}{D(\omega)}\mathcal{J}_{yz}(\omega) \quad , \\ \mathcal{J}_z^{\rm (eff)}(\omega) &= \mathcal{J}_z(\omega) + \frac{C(\omega)}{A(\omega)}\mathcal{J}_{yz}(\omega) \quad . \end{aligned} \tag{13}$$

Since eq.(12) is in the same form as eq.(4), the solution obtained from eq.(12) can be a microscopic counterpart of the mode discussed by Bohr and Mottelson. But, in the microscopic case, many normal modes are obtained from a dispersion equation and the moments of inertia depend on ω. Since the gamma vibration is the only low-lying collective mode in the negative-signature sector in the axially-symmetric and non-rotating limit, it seems natural that, among many normal modes, the gamma vibration changes its character gradually to the wobbling motion through the rotational K-mixing.

The rotational K-mixing in phonon wave functions can be measured by quadrupole-transition amplitudes associated with each phonon. They are defined as

$$T_K^{(-)}(n) = [Q_K^{(-)}, X_n^{(-)\dagger}]_{\rm RPA} \quad . \tag{14}$$

Henceforth we concentrated on the gamma-vibrational phonon and omit the index (n). Obviously $T_1^{(-)}$ is zero in axially-symmetric non-rotating nuclei and $|T_2^{(-)}/T_1^{(-)}|$ decreases as the K-mixing develops. When $< Q_2^{(+)} > \neq 0$, we can derive an analytic expression:

$$\frac{T_2^{(-)}}{T_1^{(-)}} = -\frac{2\alpha_2}{\sqrt{3}\alpha_0 - \alpha_2}\frac{C(\omega)}{D(\omega)} \quad . \tag{15}$$

where $C(\omega)$ and $D(\omega)$ are defined in eq.(10) and α_K's are the deformation parameters of a rotating potential:

$$h' = h_{\rm sph} - \sum_{K=0,2} \alpha_K Q_K^{(+)} - \hbar\omega_{\rm rot} J_x \quad . \tag{16}$$

Equation (15) produces two results. First, this analytic expression makes it possible to discuss the relation between the microscopic and macroscopic descriptions of the wobbling motion as follows. The PA-frame components of angular momentum in eq.(1) can be expressed in terms of the operators in the UR frame as[2])

$$\begin{aligned} iJ_y^{(\mathrm{PA})} &= iJ_y^{(\mathrm{UR})} - \frac{\kappa}{2\alpha_2} < J_x > Q_2^{(-)} \quad , \\ J_z^{(\mathrm{PA})} &= J_z^{(\mathrm{UR})} - \frac{\kappa}{\sqrt{3}\alpha_0 - \alpha_2} < J_x > Q_1^{(-)} \quad , \end{aligned} \tag{17}$$

where κ is a quadrupole-interaction strength. Accordingly $X_{\mathrm{W}}^{\dagger}$ can be expressed as

$$X_{\mathrm{W}}^{\dagger} = a_y iJ_y^{(\mathrm{UR})} + a_z J_z^{(\mathrm{UR})} + b_y Q_2^{(-)} + b_z Q_1^{(-)} \quad , \tag{18}$$

with

$$\begin{aligned} a_y &= \frac{x - y}{\sqrt{2 < J_x >}} \quad , \\ a_z &= -\frac{x + y}{\sqrt{2 < J_x >}} \quad , \\ b_y &= -\frac{\kappa}{2\alpha_2} < J_x > a_y \quad , \\ b_z &= -\frac{\kappa}{\sqrt{3}\alpha_0 - \alpha_2} < J_x > a_z \quad . \end{aligned} \tag{19}$$

The transition amplitudes, therefore, are calculated as

$$\begin{aligned} T_1^{(-)} &= [Q_1^{(-)}, X_W^{\dagger}]_{\mathrm{RPA}} = (\sqrt{3} < Q_0^{(+)} > - < Q_2^{(+)} >) a_y \quad , \\ T_2^{(-)} &= [Q_2^{(-)}, X_W^{\dagger}]_{\mathrm{RPA}} = -2 < Q_2^{(+)} > a_z \quad . \end{aligned} \tag{20}$$

These equations mean that the transition amplitudes have nothing to do with the second terms in eqs.(17) which assure the algebra of the PA components (see eq.(21)). Equating the ratio $T_2^{(-)}/T_1^{(-)}$ composed of eqs.(20) with eq.(15) and requiring a kind of selfconsistency between the potential and the density as

$$\frac{2\alpha_2}{\sqrt{3}\alpha_0 - \alpha_2} = \frac{2 < Q_2^{(+)} >}{\sqrt{3} < Q_0^{(+)} > - < Q_2^{(+)} >} \quad , \tag{21}$$

we obtain a relation between the microscopic quantities $C(\omega)$ and $D(\omega)$ and the macroscopic ones x and y:

$$\frac{C(\omega)}{D(\omega)} = \frac{a_z}{a_y} = \frac{y + x}{y - x} \quad . \tag{22}$$

This can be obtained also from a direct comparison of the expressions for $B(E2 : I \to I-1)$ without the aid of eqs.(17) as follows. After substituting eq.(21), eq.(15) can be rewritten in terms of the quadrupole moments quantized along the x axis as

$$\frac{T_2^{(-)}}{T_1^{(-)}} = \frac{\sqrt{3} < Q_0' > - \sqrt{2} < Q_2' >}{\sqrt{3} < Q_0' > + \sqrt{2} < Q_2' >} \frac{C(\omega)}{D(\omega)} \quad . \tag{23}$$

Therefore, according to the Marshalek's formula[5]),the $B(E2)$ takes the form:

$$\begin{aligned} &B(E2 : I_{\mathrm{W}} - (I-1)_{\mathrm{yr}}) \\ &= \frac{1}{2}(T_1^{(-)} - T_2^{(-)})^2 \\ &\propto \{\sqrt{3} < Q_0' > (C(\omega) - D(\omega)) - \sqrt{2} < Q_2' > (C(\omega) + D(\omega))\}^2 \quad . \end{aligned} \tag{24}$$

On the other hand, it is given by eq.(5) macroscopically. Consequently we obtain eq.(22) again.

The second result of eq.(15) concerns the relative sign between $T_1^{(-)}$ and $T_2^{(-)}$. Combining eq.(22) and the normalization condition on $X_{\mathrm{W}}^{\dagger}$, $x^2 - y^2 = 1$, we find that $C(\omega)/D(\omega)$ is negative definite. Besides, the factor including α_K's in eq.(15) can be expressed as a function of $\gamma^{(\mathrm{pot})}$ [ref.6)]:

$$\frac{2\alpha_2}{\sqrt{3}\alpha_0 - \alpha_2} = -\frac{\sin \gamma^{(\mathrm{pot})}}{\sin(\gamma^{(\mathrm{pot})} + 60°)} \quad , \tag{25}$$

where the sign of γ conforms to the Lund convention and is opposite to the convention adopted in ref.6). Consequently the sign of $T_2^{(-)}/T_1^{(-)}$ is determined uniquely by $\gamma^{(\mathrm{pot})}$. This sign has never been discussed up to now although it has important physical meanings. The most interesting one is the ω_{rot}-dependence of $B(E2 : I_{\mathrm{W}} - (I-1)_{\mathrm{yr}})$. Since $|T_2^{(-)}/T_1^{(-)}|$ decreases as ω_{rot} increases, the $B(E2)$

is a decreasing function of $\omega_{\rm rot}$ or spin if $T_2^{(-)}/T_1^{(-)}$ is positive while it is an increasing function if the ratio is negative as depicted in fig.1 (see eq.(24)). The former and the latter cases are expected to be realized in nuclei with $\gamma^{(\rm pot)} < 0$ and $\gamma^{(\rm pot)} > 0$, respectively. A numerical example of the former can be found in ref.7). The transitions between the states with spin $I_{\rm W}$ and with $(I+1)_{\rm yr}$, whose direction depends on the excitation energy of the wobbling, can also be considered. In these cases the $\omega_{\rm rot}$-dependence is opposite since they are proportional to $(T_1^{(-)} + T_2^{(-)})^2$. In addition, the phase rule of $T_K^{(-)}$'s determines the properties of quasiparticle-vibration-coupling wave functions of odd-A nuclei[8]).

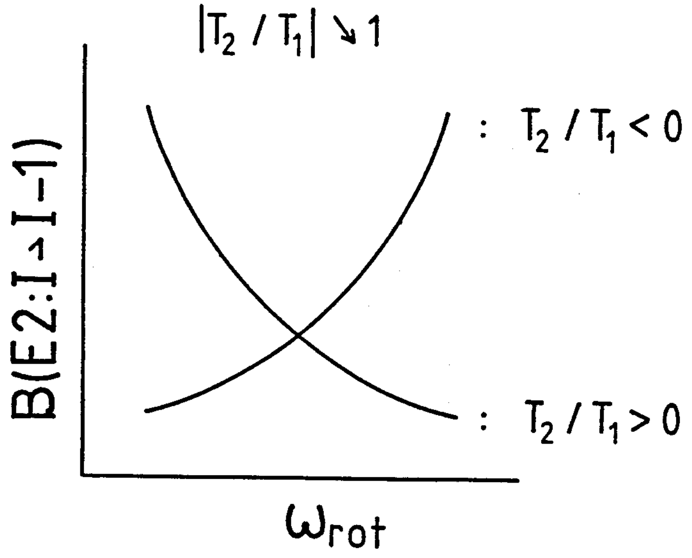

Fig.1. A schematic drawing of the behavior of $B(E2 : I_{\rm W} \rightarrow (I-1)_{\rm yr})$ as a function of $\omega_{\rm rot}$.

In summary, we have studied the character change of the gamma-vibrational excitation with $r = -1$ in rotating triaxial nuclei to the wobbling motion. By evaluating the transition amplitudes connecting it to the yrast band, the relation

between the microscopic and macroscopic descriptions of the nuclear wobbling motion and the γ-dependence of the rotational behavior of the $B(E2 : \Delta I = 1)$ values have been clarified.

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