Prog. Theor. Phys. Vol. 77, No. 6, June 1987, Progress Letters

Signature Dependence of $M1$ and $E2$ Transitions in Rotating Triaxial Odd-A Nuclei

Masayuki MATSUZAKI, Yoshifumi R. SHIMIZU* and Kenichi MATSUYANAGI

Department of Physics, Kyoto University, Kyoto 606
**Department of Physics, Kyushu University, Fukuoka 812*

(Received December 18, 1986)

Effects of both the static and the dynamic triaxial deformations on the signature dependence of $B(M1)$ and $B(E2)$ in odd-A nuclei are studied by applying the RPA formalism based on the rotating shell model to odd-A nuclei. Numerical examples are presented for ^{165}Lu and ^{157}Ho. The observed $M1$ properties of the high-spin region of ^{165}Lu are well reproduced, while the extremely strong signature dependence of $B(E2)$ in ^{157}Ho is not reproduced by the calculation.

In recent years, strong signature dependence of the $B(M1)$ and $B(E2)$ values has been observed[1),2)] in the $\Delta I=1$ transitions between high-spin states in odd-A nuclei. These experimental data have been discussed in connection with the occurrence of triaxial deformations.[3),4)] The theoretical calculations have been done mainly in terms of the particle-rotor model.[3)~5)] In addition to the effects of the static triaxial deformations, those of the γ-vibrations have also been discussed, within the framework of the macroscopic Bohr-Mottelson model, in some cases.[6),7)]

The purpose of our work is to develop, on the basis of the Rotating Shell Model (RSM), a microscopic description of odd-A high-spin states along the line parallel to the phenomenological particle-rotor model. In this letter, we report a few typical results obtained by applying the RPA formalism based on the RSM[8)] to odd-A nuclei. One of the advantages of our approach is that it can easily be applied to high-spin states with many aligned quasiparticles, while it has a limitation that the γ-vibrations and the wobbling modes are treated within the small amplitude approximation. The main steps of our microscopic approach are as follows:

1) We construct a diabatic quasiparticle representation for the rotating triaxially deformed potential that is obtained by transforming the single-particle potential of the Nilsson plus BCS form into a uniformly rotating frame of reference with a given value of the rotational frequency $\omega_{\rm rot}$. This step provides us with the diabatic basis for the RSM.

2) The residual interactions consisting of the monopole-pairing and the doubly-stretched quadrupole forces are treated by means of the RPA in the rotating frame,[8)] and normal modes of vibration are determined.

3) For odd-A nuclei, the couplings of the aligned quasiparticles with the γ-vibrations in the rotating frame are treated in a way similar to the traditional quasiparticle-phonon coupling models.[9)] The γ-vibrations are taken into account up to the two-phonon states, and the Pauli principle effects stemming from the two-quasiparticle structure of the γ-vibration are neglected.

4) We extend Marshalek's treatment[10)] of the Nambu-Goldstone modes, $\Gamma^\dagger$ and Γ, of the RPA to odd-A nuclei. Namely, we make the following replacement,

$$\left.\begin{aligned} \Gamma^\dagger &\to \frac{1}{\sqrt{2I}}(\hat{I}_- - \hat{J}_-^{(\mathrm{qp})}), \\ \Gamma &\to \frac{1}{\sqrt{2I}}(\hat{I}_+ - \hat{J}_+^{(\mathrm{qp})}), \end{aligned}\right\} \tag{1}$$

where $\hat{I}_\pm$ represents the total angular momentum and $\hat{J}_\pm^{(\mathrm{qp})}$ the quasiparticle angular momentum. This ansatz is the most crucial point of our approach. The motive of this replacement is to develop a microscopic model in which the state vectors are constructed, as in the particle-rotor model, in a direct-product form of the rotational and the internal wave functions. A similar ansatz was previously adopted by Hara and Kusuno[11)] for the case $\omega_{\mathrm{rot}}=0$. Once the replacement (1) is admitted, it is straightforward to extend Marshalek's procedure[10)] for constructing the state vectors in the laboratory frame to odd-A nuclei.

By means of the above procedure, we obtain effective electromagnetic operators acting on the internal wave functions of odd-A nuclei as follows:

M1 transitions with $\Delta I=-1$

$$\hat{\mu}_{-1}=(g_l-g_{\mathrm{RPA}})\,\hat{l}_{-1}^{(\mathrm{qp})}+(g_s^{(\mathrm{eff})}-g_{\mathrm{RPA}})\,\hat{s}_{-1}^{(\mathrm{qp})} + \text{vibrational terms}, \tag{2}$$

where $\hat{\boldsymbol{l}}^{(\mathrm{qp})}$ and $\hat{\boldsymbol{s}}^{(\mathrm{qp})}$ denote the orbital and spin angular momenta of quasiparticles, and the vibrational contributions consist of the linear terms with respect to the RPA phonon creation and annihilation operators (see Ref. 10)). The effective g-factor of the RPA vacuum state, g_{RPA}, is given by

$$g_{\mathrm{RPA}}=\frac{\langle\hat{\mu}_x\rangle}{\langle\hat{J}_x\rangle}, \tag{3}$$

where the expectation values are taken with respect to the RPA vacuum states.

E2 transitions with $\Delta I=-1$

$$\frac{1}{i}\hat{Q}_{2-1}=-\sqrt{\frac{3}{2}}\langle Q_0\rangle\frac{\hat{J}_z^{(\mathrm{qp})}}{I}+\langle Q_2\rangle\left(2\frac{i\hat{J}_y^{(\mathrm{qp})}}{I}+\frac{\hat{J}_z^{(\mathrm{qp})}}{I}\right) + \text{vibrational terms}, \tag{4}$$

where the quadrupole operator $\hat{Q}_{2\mu}$ is quantized along the rotation axis (the x-axis), and $\langle Q_K\rangle$ $(K=0, 2)$ represent the expectation values of the quadrupole operators Q_{2K} whose quantization axis is the z-axis. The vibrational terms consist of the linear terms with respect to the RPA phonon operators. When the triaxial deformation is small, Eq. (4) can be rewritten in a good approximation as

$$\frac{1}{i}\hat{Q}_{2-1}=\left\{-\sqrt{\frac{3}{2}}\langle Q_0\rangle+\langle Q_2\rangle\left(1+2(-1)^{I-j}\left|\frac{\Delta E}{\hbar\omega_{\mathrm{rot}}}\right|\right)\right\}\frac{\hat{J}_z^{(\mathrm{qp})}}{I} + \text{vibrational terms}, \tag{5}$$

where ΔE denotes the signature splitting of the quasiparticle energies. For the quasiparticles associated with the j-shell, ΔE may be written as

$$\Delta E \equiv E\left(\alpha=\frac{1}{2}\right)-E\left(\alpha=-\frac{1}{2}\right)=(-1)^{j+1/2}|\Delta E|\,, \tag{6}$$

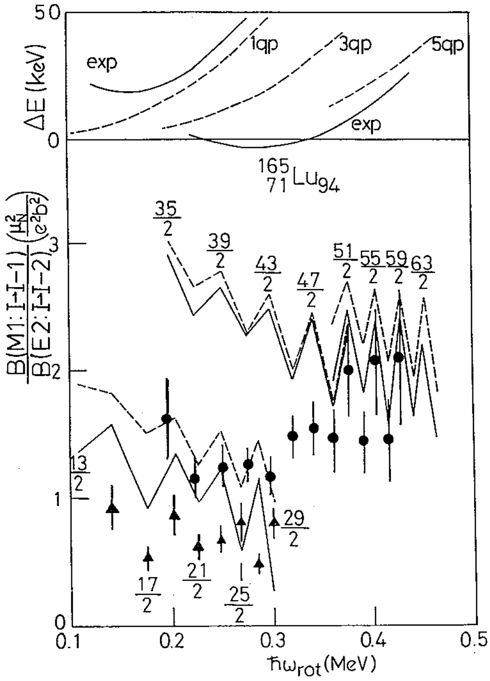

Fig. 1. The ratios $B(M1; I\to I-1)/B(E2; I\to I-2)$ plotted as a function of $\omega_{\rm rot}$. The solid triangles and the solid circles with error bars denote the experimental data.[1] The solid (broken) lines represent the results of calculation with (without) taking into account the couplings with the γ-vibrations. The experimental values for $I=31/2$ and $33/2$ lying in the band crossing region are omitted from the figure. The triaxial deformation parameters are assumed to be $\gamma_0=18°$, $10°$ and $0°$ for the one-, three- and five-quasiparticle bands, respectively. Note that our definition of the sign of γ_0 is opposite to the Lund convention.[3] Other parameters of calculation are: $\beta=0.21$, $g_s^{(\rm eff)}=0.7g_s^{(\rm free)}$, $\Delta_p=1.20$ MeV, $\Delta_n=1.16$ MeV for the one-quasiparticle band, $\Delta_p=1.18$ MeV, $\Delta_n=0.72$ MeV for the three-quasiparticle band, and $\Delta_p=1.18$ MeV, $\Delta_n=0$ for the five-quasiparticle band. In the upper portion of this figure, calculated values for the signature-splittings of the quasiparticle energies are compared with the experimental ones.

where j is related to the signature α by $j=\alpha+$even. The expression (5) reduces to that of Hamamoto[3] when $j=1/2$, because in this case the factor $(-1)^{I-j}|\Delta E/\hbar\omega_{\rm rot}|$ becomes $(-1)^{I-1/2}$. Note that the factor multiplying $\langle Q_2\rangle$ in Eq. (5) always takes larger values for the favoured states (whose $I=j+$even) than for the unfavoured states (whose $I=j+$odd). Note also that the ratio $|\Delta E/\hbar\omega_{\rm rot}|$ is smaller than unity in general.

Below we present typical results of numerical calculations for ^{165}Lu and ^{157}Ho, for which most detailed experimental data are available. In these calculations, the same static triaxial deformation parameters as in Refs. 3) and 4) are used for the sake of comparison with these works, except for the five-quasiparticle aligned band of ^{165}Lu where $\gamma_0=0°$ is assumed. For the sake of reference, we also mention that the static triaxial deformations which we calculated by using the method of Ref. 12) are $\gamma_0=0°\sim8°$ for the one-quasiparticle band, $\gamma_0=6°\sim11°$ for the three-quasiparticle band and $\gamma_0\approx0°$ for the five-quasiparticle band. (Note that our definition of the sign of γ_0 is opposite to the Lund convention.[3]) These values of γ_0 smoothly change as a function of $\omega_{\rm rot}$ within individual bands. The principle of fixing other parameters entering in the calculation is the same as in Ref. 8), and the details will be given in a forthcoming paper.

Figure 1 shows the ratios $B(M1; I\to I-1)/B(E2; I\to I-2)$ for ^{165}Lu as a function of ω_{rot}. The solid (broken) lines represent the ratios calculated with (without) taking the couplings with the γ-vibrations into account. The observed rotational bands may be roughly divided into three groups according to the number of aligned quasiparticles. The first group ($15/2 \lesssim I \lesssim 29/2$) involves the aligned quasiparticle A_p or B_p. The second ($35/2 \lesssim I \lesssim 51/2$) involves the quasiparticle configuration $A_pA_nB_n$ or $B_pA_nB_n$. The third ($I \gtrsim 59/2$) involves $A_pA_nB_nC_nD_n$ or $B_p A_nB_nC_nD_n$. Here A_p, B_p and A_n, B_n, C_n, D_n are the familiar notations[1] denoting the aligned quasiparticle states associated with the $\pi h_{11/2}$ and $\nu i_{13/2}$ shells, respectively. Note that we obtain the crossing between the second and the third configurations at $\hbar\omega_{rot} \simeq 0.4$ MeV in good agreement with the suggestion from the experiment.[1] The interactions between the two configurations are neglected in our calculation with the diabatic approximation, although the experimental data indicate that these are rather strong. The selfconsistently calculated neutron pairing gaps are $\Delta_n = 0.72$ MeV at $\hbar\omega_{rot} = 0.2$ MeV for the three-quasiparticle band and $\Delta_n = 0$ for the five-quasiparticle band,*) when the pairing-force strength G is fixed at the ground state to reproduce the odd-even mass difference. We see in Fig. 1 that the signature dependence is well reproduced especially in the highest-spin region. Another interesting feature of Fig. 1 is that the ratio increases when the $i_{13/2}$ neutrons align. This trend is easily understandable from Eqs. (2) and (3), since the aligned $i_{13/2}$ neutrons decrease the value of g_{RPA}.[13)~15)] The calculated values of g_{RPA} are $0.29 \sim 0.27$ for the one-quasiparticle band, $-0.12 \sim -0.04$ for the three-quasiparticle bands, and $-0.05 \sim -0.04$ for the five-quasiparticle band. These values of g_{RPA} smoothly change as a function of ω_{rot} within individual bands.

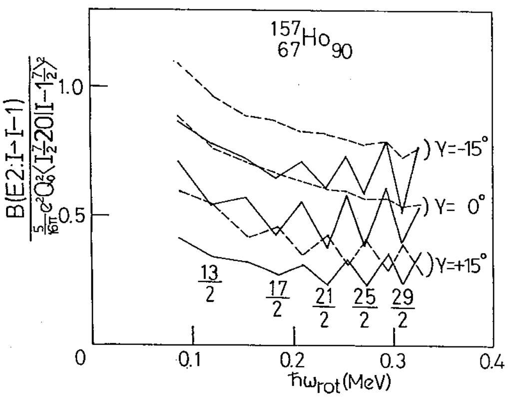

Fig. 2. The calculated values of $B(E2; I \to I-1)$ divided by $(5/16\pi)e^2\langle Q_0\rangle^2\langle I, 7/2, 2, 0|I-1, 7/2\rangle^2$. The three cases with different γ_0 values ($\gamma_0 = \pm 15°, 0°$) are displayed. The solid (broken) lines show the results with (without) taking the couplings with the γ-vibrations into account. The deformation parameters used are $\beta = 0.20$, $\Delta_p = 1.21$ and $\Delta_n = 1.25$ MeV.

Figure 2 shows the calculated values of $B(E2; I \to I-1)$ for ^{157}Ho divided by $(5/16\pi)e^2\langle Q_0\rangle^2\langle I, 7/2, 2, 0|I-1, 7/2\rangle^2$. It is seen that the signature dependence originating from the couplings with the γ-vibrations is stronger than that from the static triaxial deformations. Consequently, the $B(E2; I \to I-1)$ from the favoured states (whose $I = j +$even) become always larger than those from the unfavoured states (whose $I = j +$odd), in agreement with the experimental data.[2)] On the other hand, the calculated signature dependence of $B(E2)$ is smaller in magnitude than experimental data. Also, the large experimental values[2)] of the ratio $B(E2; I \to I-1)/B(E2; I \to I-2)$ could not be reproduced. In this connection, we note

*) As shown in Ref. 8), the pairing gaps depend only weakly on ω_{rot} within each rotational band defined by the diabatic quasiparticle representation. Therefore, we used constant pairing gaps for each band in Fig. 1.

that the calculated values of the factor $\Delta E/\hbar\omega_{rot}$ are 0.43, 0.08 and -0.05 for $\gamma_0=15°$, $0°$ and $-15°$, respectively, at $\hbar\omega_{rot}=0.2$ MeV in ^{157}Ho, which are considerably smaller than unity. Thus, the signature dependence originating from the static triaxial deformations is significantly suppressed in this nucleus.

The results presented above are in qualitative agreement*) with those of Refs. 3), 4), 6), 7) and 15) except for 1) the case $\gamma=15°$ in Fig. 2**) and 2) the highest-spin region of ^{165}Lu, for which the calculation including the five aligned quasiparticles was carried out for the first time. The microscopic model outlined in this paper may be regarded as a kind of extension of the particle-rotor model, since the basis of the intrinsic state vectors is determined by the RSM as a function of ω_{rot}. One of the merits of this approach is that multi-nucleon-aligned bands can be easily treated, whereas, in the conventional particle-rotor model, the treatment of such bands becomes increasingly difficult with increasing number of aligned nucleons. The results of calculation presented here show the feasibility of our approach. A systematic calculation for many high-spin data is being performed, and will be reported in a forthcoming paper in which the details of formulation will also be given.

The computer calculation for this work has been financially supported by Research Center for Nuclear Physics, Osaka University.

---

1) P. Frandsen et al., Phys. Lett. **B177** (1986), 287.
2) G. B. Hagemann et al., Nucl. Phys. **A424** (1984), 365.
3) I. Hamamoto, *Proc. Niels Bohr Centennial Conf. on Nuclear Structure, Copenhagen 1985*, ed. R. Broglia, G. B. Hagemann and B. Herskind (North-Holland, 1985), p. 129, and references therein.
4) I. Hamamoto and B. Mottelson, Phys. Lett. **167B** (1986), 370.
5) F. Dönau and S. Frauendorf, *Proc. Conf. on High Angular Momentum Properties of Nuclei , Oak Ridge 1982*, ed. N. R. Johnson (Harwood, New York, 1983), p. 143.
6) A. Ikeda, Nucl. Phys. **A439** (1985), 317.
7) N. Onishi, I. Hamamoto, S. Åberg and A. Ikeda, Nucl. Phys. **A452** (1986), 71.
8) Y. R. Shimizu and K. Matsuyanagi, Prog. Theor. Phys. **70** (1983), 144; **72** (1984), 799.
9) L. S. Kisslinger and R. A. Sorensen, Rev. Mod. Phys. **35** (1963), 853.
10) E. R. Marshalek, Nucl. Phys. **A275** (1977), 416.
11) K. Hara and S. Kusuno, Nucl. Phys. **A245** (1975), 147.
12) Y. R. Shimizu and K. Matsuyanagi, Prog. Theor. Phys. **71** (1984), 960.
13) M. Sano, T. Takemasa and M. Wakai, *Proc. Int. Conf. on Nuclear Moments and Nuclear Structure, Osaka 1972*, ed. H. Horie and K. Sugimoto, J. Phys. Soc. Jpn. **34** Suppl. (1973), 365.
14) S. Frauendorf, Phys. Lett. **100B** (1981), 219.
15) Y. S. Chen, P. B. Semmes and G. A. Leander, Phys. Rev. **C34** (1986), 1935.

*) According to Ref. 7), the couplings with the γ-vibrations enhance the $B(E2; I\to I-1)$ from the unfavoured states over those from the favoured states. We confirmed that a similar result is obtained also in our approach if the chemical potential lies, as in their case, below the $K=1/2$ states associated with the high-j orbit. Since the chemical potential lies near the $K=7/2$ orbit in the case ^{157}Ho, there is no contradiction between the results shown in Fig. 2 and that of Ref. 7).

**) The reason for the disagreement between our result and that of Ref. 3) in the $\gamma=15°$ case of ^{157}Ho is not clear.